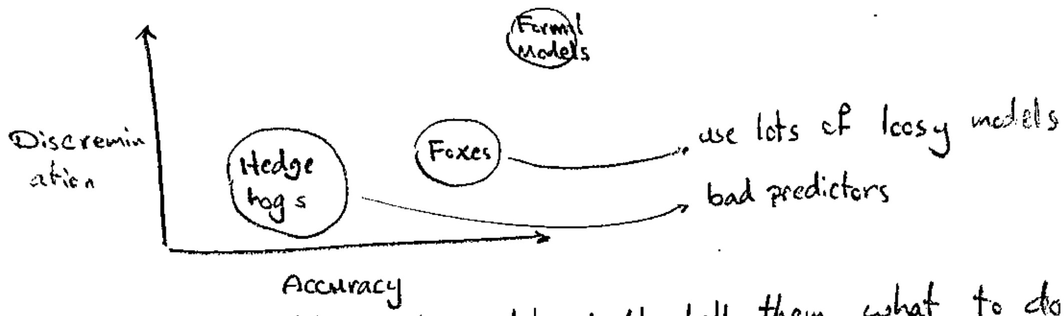
# Model Thinking

( Page - Ann Arbor )

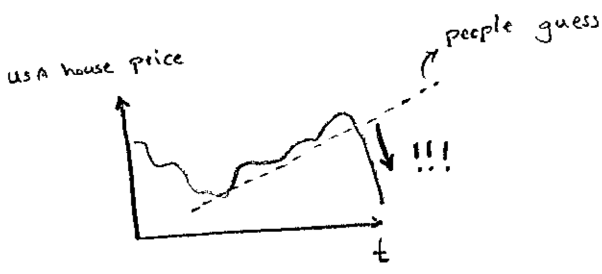
* why Models?
    - Intelligent Citizen of the world
    - Clearer Thinker
    - Understand and Use Data
    - Decide, Strategize, and Design

* Intelligent Citizens
    - "All models are wrong, but they are useful" → Abstraction, simplification
    - Language of Jargeon
    - All the ideas everyone should know → Tie to the mast
        - ☑ Odyssey → hear the sirens
        - Cortez → burning ships not to retreat
        - Adjudicate between two opposite advices
            - ↳ Models clarify the condition for each of advices to be true
    - Models are better than we are
        - ☑ prediction




- Smart people use models, but models don't tell them what to do
    - ↳ use models only as guidelines

- Models are fertile
    - use in other domains
- Make us humble
    - show us the full dimensionality of data




- Many Model Thinkers know Best

* Clearer Thinker
    - Name the parts → Relevant Features
    - Identify Relationships

- Work Through Logic → Intuitions made mistakes

  ☑ rim around the earth with 1m loose space

$P_{earth} = 25{,}000$ miles

$P_{rim} = 25\,000$ miles $+ 6.28$ m !!!

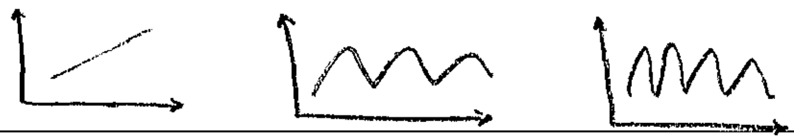
$P_E = 2\pi R_E = 25000$ mil

$P_{rim} = 2\pi(R_E + 1) = P_E + 6.28$ !

- Inductively Explore
- Understand Class of outcome → Equilibrium / Cycle / Random / Complex
- Identify Logical Boundaries
  ↳ finding the condition under which one statement hold
- Communicate

* Use and Understand Data

1. Understand Basic patterns



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2. Predict Points
3. Produce Bounds → ☑ inflation  1 year : 1.2 %
   10 years : [0, 3] %
4. Retrodiction : Use model with data to predict the past
   - why? → what do we think the past was like? → no data
     → How good is the model we have → verify model
5. Predicting Other stuff
   - Make model for one reason, gives you other systems too.
   - Predict the things that we ignore to see → ☑ discovery of Neptune by model of solar system
6. To Inform Data Collection
   - to help getting relevant data
7. Estimate Hidden Parameters
8. Calibration → Make model first and then fit it to real data

* Decide, Strategize, and Design

1. Decision Aids
   - ↳ The Monty Hall Problem
   - ↳ Real Time
2. Comparative Statics : Move from one equilibrium to another
3. Counterfactuals : What if we didn't make a decision? what could happen?
4. Identify & Rank Levers
5. Experimental Design : How do I run the best possible experiment
   ↳ most informative
6. Institutional Design : How to make systems closer to ideals
7. Help Choose Among Policies and Institutions ⟶ ☑ decide : Market / Democracy / Beaucracy } ?

* <u>Sorting</u> and Peer Effects

↳ Homophily

- Feel like, act like, think like people we are hanging out with → sorting
- Schelling (Tipping) Model → Racial Segregation → segregation model
- Granovetter Model → People willingness to participate in some sort of collective behavior
  ☑ riot, political uprising, social movement
- Standing Ovation Model → Model of peer effect when you change your behavior to match that of others around you
- Identification → Group of Similar People → sort? they chose to hang out with each other
  ↘ peer effect? they become like each other by time

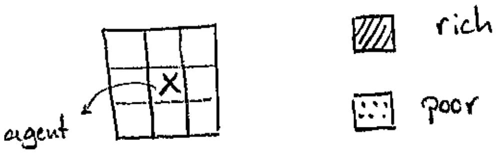
- Equation Based Model ✗
  Agent Based Model → individual ☑ ppl, firm, countries, organization
  → behaviors → rules they follow
  ↳ optimized? → game theory
  → outcome → result of agents with behaviors

# * Schelling's Spatial Segregation

- racial segregation / by income
- agent based model → agents : people
  - ↘ behaviors : what rules do they follow → threshold
  - ↘ outcome : aggrgation (add 'em up)
- about people choosing where to live
  - ↳ people decision : should I stay here or should I move?
  - ↳ people living in checker board

    color : race / income

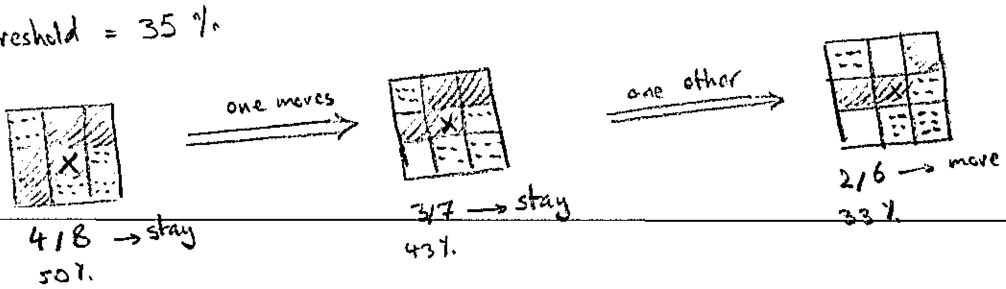
    blank : no neighbor




☑  3/7 like her

according to her threshold → stay / → move

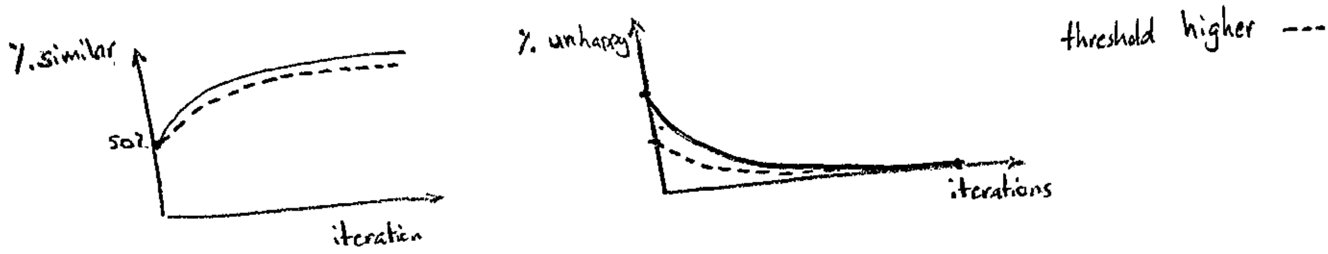
☑ threshold = 35%

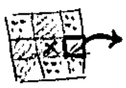



Computer Modeling : Netlogo

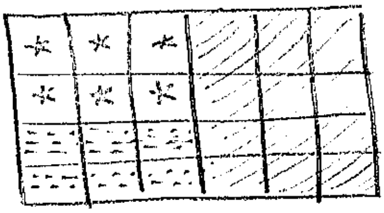
- Micro level → threshold

  Macro level → segregation → even if people are tolerant (threshold ↓)




- Threshold too high ⇒ no equibilirium
- Micromotive ≠ Macro behavior
- Tipping → people who cause other people to leave (= go beyond Threshold
  - Exodus Tip : leaving one cause another to leave 
  - Genesis Tip : moving in cause another to leave 

- Measure of Dissimilarity



rich

poor

50/50

each block = 10 people

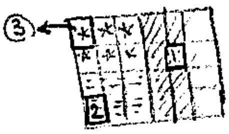
$R$ = rich = 12 × 10 + 6 × 5 = 150

$P$ = poor = 90

$\frac{p}{P}$ = % poor people in that block

$\frac{r}{R}$ = % rich people in that block

$\Longrightarrow \frac{1}{2}\left|\frac{r}{R} - \frac{p}{P}\right|$ = index of dissimilarity



① $\left|\frac{10}{150} - \frac{0}{90}\right| = \frac{1}{15}$

② $\left|\frac{0}{150} - \frac{10}{90}\right| = \frac{1}{9}$

③ $\left|\frac{5}{150} - \frac{5}{90}\right| = \frac{1}{45}$

→ overall: 6 × (50/50) + 6 × (poor) + 12 × (rich) =

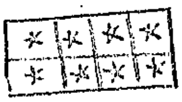
$6 \times \frac{1}{45} + 6 \times \frac{1}{9} + 12 \times \frac{1}{15} =$

$\frac{72}{45}$ ⟶ divide by 2: $\frac{72}{90} = 80\%$

① $\left|\frac{10}{40} - \frac{0}{40}\right| = \frac{1}{4}$

② $\left|\frac{0}{40} - \frac{10}{40}\right| = \frac{1}{4}$

→ overall: $4 \times \frac{1}{4} + 4 \times \frac{1}{4} = 2$ — Perfectly segregated



$\left|\frac{5}{40} - \frac{5}{40}\right| = 0$

→ overall: $8 \times 0 = 0$ — Perfectly mixed

① $\left|0 - \frac{1}{2}\right| = \frac{1}{2}$

② $\left|\frac{1}{2} - 0\right| = \frac{1}{2}$

③ $\left|\frac{1}{2} - \frac{1}{2}\right| = 0$

→ overall: $1 \times \frac{1}{2} + 1 \times \frac{1}{2} + 2 \times 0 = 1$

↳ divide by 2: 0.5 = 50%

- index ∈ [0, 1] ⟶ [segregated, mixed]

## * Granovettor's Model

- N people ⟶ every one has a threshold $T_j$ ⟶ he join the cause if $T_j$ other people join the cause.
- $T_j = 0$ ⟶ initiator!

☑ $T = \{0, 1, 2, 2, 2\}$ ⟶ every one involved

$T = \{1, 1, 1, 2, 1\}$ ⟶ noone ⟶ ave = 1.4 ⇒ noone to get it started

$T = \{0, 1, 2, 3, 4\}$ ⟶ everyone ⟶ ave = 2.5 ⇒ people really don't like it but do it!

- Collective Action More Likely if:
  - Lower thresholds
  - More Variation in thresholds → average not useful
    ↘ you'd better know the distribution

* Standing Ovation Model

- Peer Effect + Information
- T: Threshold to Stand
  Q: Quality
  S: Signal = Q + Error

⟹ Stand if S > T (initial rule)
Stand if more than X % stand (subsequent rule)

- Claims:
  - Higher Q ⟹ more people stand ↑Q + error > T
  - Lower T ⟹ more people Q + error > T↓
  - Lower X ⟹ more people ⤳ peer effect
    ↳ Big X: need more security
    Small X: get excited easily
  - E → Error ⤳ in the sense of quality
    ↘ Diversity ⤳ in the sense of people perception of the quality

☑ 1000 ppl, T = 60, Q = 50 → no oviation condition

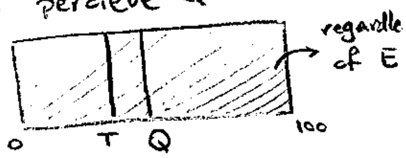
E ∈ [-15, +15] ⤳ S ∈ [35, 65] → small ovation

E ∈ [-50, +50] ⤳ S ∈ [0, 100] → 40% ppl stand up → depends on X

if Q < T then more variation in E ⟹ more pple Q + E↑ > T

E → Audience → diverse
↘ unsophisticated
E → performance → multidimensional
↘ complex

☑ if Q > T → more than the half of audience percieve Q above their T so they stand up

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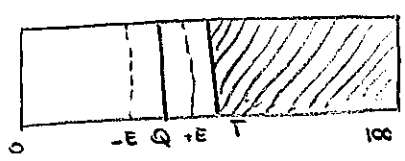
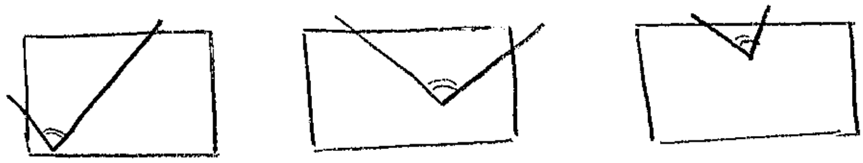


• Oviation Advanced

- you don't see people behind you



→ size of cone

↳ people in front are like celebrities

↳ people in back are like academics

- people go to shows in pairs or groups

• How to cause standing ovations
  - Higher Quality
  - Lower Threshold
  - Larger Peer Effect
  - More Variation
  - Use Celebrities
  - Big Groups

☑ Collective Actions, Academic Performance, Urban Renewal, Fitness / Health, Online Course

* Identification

• People similar → homophily or peer effect?

☑ Sorting : pop 1 : A A B B A A / B B A B B A

☑ peer effect : pop 2 : A A B(A) B(A) A A / B B A(B) B B A(B)

⇒ Result : A A A A A A / B B B B B B !

• Can't be told by snapshot → Need process (dynamic data)
  - Look for moving → sorting
    changing → peer effect

* Aggregation :

- "More is Different" → Phillip Anderson
  ↘ Connection of elements have different properties

  ☑ 1 molecule of water is not wet

  1 neuron cannot bring concicusness → emergent properties of a system

- Actions → Central Limit theorem
- Single Rule
- Family of Rule
- Preferences → Collective Choices

- Predict Points

  Understand Data

  Understand Patterns → ☑ Recurrent Patterns, Game of Life

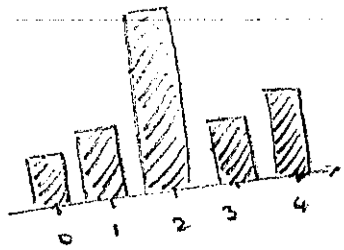
  Understand Class of Outcome → Equilibria, Pattern, Chaos (Random), Complex

  Work through logic → paradoxes, problems

* Probability Distribution

- what people decide to do?

  ☑ go for a walk in 4 person family

  | # ppl: | 0 | 1 | 2 | 3 | 4 | |
  |---|---|---|---|---|---|---|
  | | 10% | 15% | 40% | 15% | 20% | → $\Sigma = 100\%$ |

- Centeral Limit Theorem

  ↳ if we add up the decisions independantly, the result will be Gaussian.

  ☑ Flip Coin 2 times

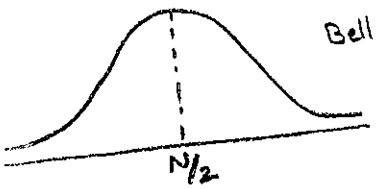
  | | |
  |---|---|
  | TT | 1/4 |
  | TH, HT | 1/2 |
  | HH | 1/4 |

  4 times

  | | |
  |---|---|
  | 0 H | 1/16 |
  | 1 H | 4/16 |
  | 2 H | 6/16 |
  | 3 H | 4/16 |
  | 4 H | 1/16 |

  Bell Curve

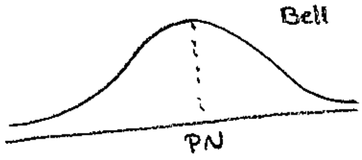
  ☑ Flip coin N times

  mean = expected val = $\frac{N}{2}$

- Binomial Distribution

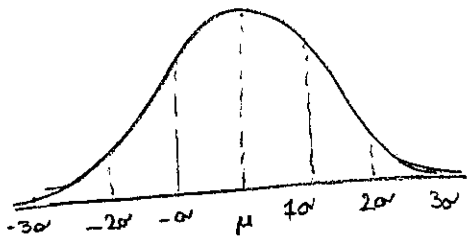
  - mean = $pN$

  not a 50-50 dicision → p: prob of doing a thing

  for enough large N

- σ = standard deviation → how far spread out that a curve is




- ±1 sd → 68%
- ±2 sd → 95%
- ±3 sd → 99.75%

☑ μ=100 , σ=5

↳ we should be ready to see 85 to 115 people by 99% chance

- S.D. in binomial dist $(p=\frac{1}{2}) = \frac{\sqrt{N}}{2}$

Mean in binomial dist $(p=\frac{1}{2}) = \frac{N}{2}$

☑ N = 100

↳ μ=50 , σ=5 → 68% of outcomes will be between 45 to 55

↳ if we flip a coin 100 times there is 68% chance that we get [45, 55] heads.

↳ we almost never get the head count under 35 or over 65 (0.25% chance)

- S.D. in binomial dist $= \sqrt{P(1-P)\,N}$

Mean in binomial dist = PN

☑ Boeing 747 : 380 seats

90% show up rate

Sell 400 Tickets !!

assume: decisions of passengers are independant. → not true: snowy weather

$\mu = 400 \times 0.9 = 360$

$\sigma = \sqrt{0.9 \times 0.1 \times 400} = 6$

68% of time → [354, 366]

95% of time → [348, 372]

99.75% of time → [342, 378] → we won't overbooked

- Centeral Limit Theorem (Formally)

Add Random Variables

↳ independant

↳ finite variance → bounded possible range

Sum to a Normal distribution

* Six Sigma → to have fewer quality errors → make production more predictable

- By Motorolla
- 3.4 in a million by range [μ-6σ, μ+6σ]

☑ Average Banana Sale : 500 lbs

S.D : 10 lbs

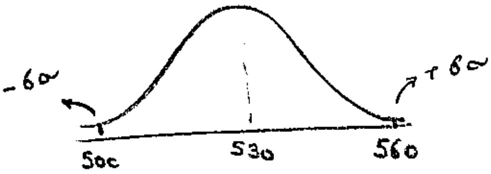
if we stock 560 lbs we never run out of bananas

☑ Required Metal Thickness : 500 - 560 mm

set averge to 530 mm

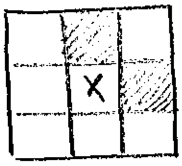
six sigma : $6\sigma = 560 - 530 \rightarrow \sigma = 5$

we should lower $\sigma$ to 5 → contineous improvement




## * Game of Life

- shows how things aggregate
- by John Conroy
- each cell → 8 neighbors → Alive ▨ Dead □

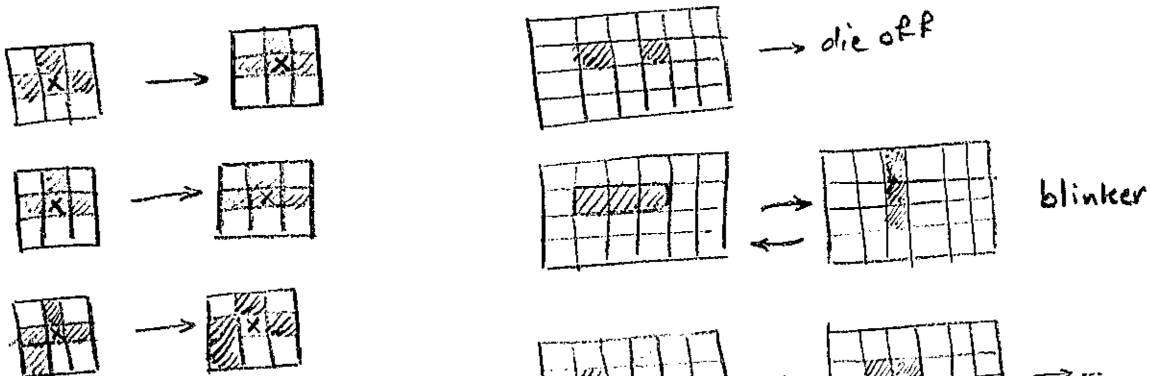


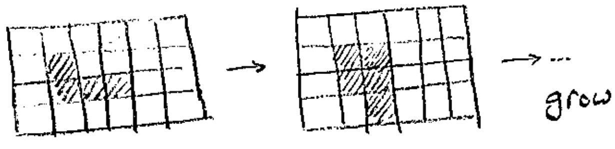
- rules → if dead can be alive if exactly three neighbors are alive.
  - if alive will be dead if fewer than two neighbors are alive → boredom
  - if alive will be dead if more than three neighbors are alive → suffocation

off —3 neigh. on→ on

on —2 or 3 neigh. on→ on (otherwise off)

☑




☑ Netlogo → Beacon 

Figure 8 

F-pimento  → produce gliders

simple glider  → shift each 4 steps

- Self Organization : Patterns appear without designer
- Emergance : Functionalities appear ☑ gliders, glider guns, counters, computers
  - ↳ from neuron to cognition

- Logic Right: simple rules produce incredible phenomena

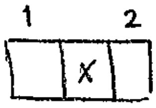


# * Celluar Automata

- 1D celluar automata

↳ 1D grid → only 2 neighbor → simple
↘ exhustive study available
↘ visualization → during time


↳ problem




| | | |
|---|---|---|
| — | — | — |
| — | — | o |
| — | o | — |
| — | o | o |
| o | — | — |
| o | — | o |
| o | o | — |
| o | o | o |

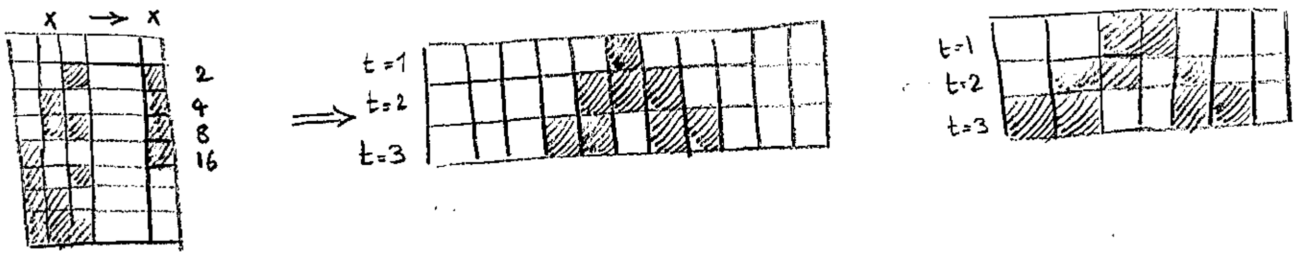
→ rule: next state regarding current Configuration (lookup table) → outcome: fixed point, randomness, alternation, Complexity

↳ naming

| | X | | | X | |
|---|---|---|---|---|---|
| — | — | — | → | □ | 1 |
| — | — | o | | □ | 2 |
| — | o | — | | □ | 4 |
| — | o | o | | | 8 |
| o | — | — | | | 16 |
| o | — | o | | | 32 |
| o | o | — | | | 64 |
| o | o | o | | | 128 |

→ get a binary Code for rules ... rule #0 to rule # 255

☑ rule # 30




# * Aggregation of Preferences

- Preferrences
  - Through Revealed Action → give them choice → which do you prefer ove the oth
    - ↳ Overall Pref ⟹ complete listing of someone's preference
      - ↳ pref. ordering → ranking of whole set → usually inside a class
        - ☑ ranking of fruits
  - How many pref. ordering are there?
    - ↳ Binary Pref → ">" or "<" sign → $2^{\#\text{ of items}}$

- Transitive Preferences

  - A > B
    B > C
    A < C → don't make sence → A > C
    ⇒ Should Satisfy transitivity RATIONALLY

  - so rational preferrences → those which satisfy transitivity

  - How many Rational? → (# of choices)!

- Aggregation : what's the collective pref.?

  - A > B > C
    B > A > C
    A > B > C
    ---
    A > B > C
    (A > B → voting; > C ↳ obvious)

  - A > B > C
    B > C > A
    C > A > B

    pairwise vote →

    C > A ②,③
    C < B ①,② } ⇒ B > A
    A > B ①,③
    ---
    C > A > B > C

    every single person : rational
    collective pref. : irrational
    ↳ paradox of aggregation
    Condorcet Paradox

- Condorcet Paradox : random vote

  people might wanna vote STRTIGECALLY → manipulate agenda
  → lie about pref.
  → mispresent pref.

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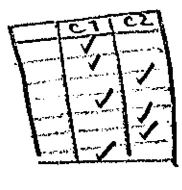
* Decision Models

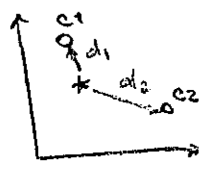
- Normative : makes us better at deciding → make us better thinker
- Positive : predict behavior of others

☑ Normative : Schol? Job? Insurance? Drive or Fly? Computer? Investments? House?

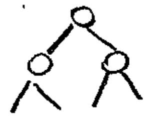
Positive : Policy Choice
Nominations
Platforms
Investments
Tech. Choice

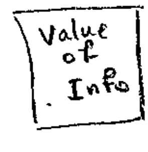
- Classes of Decision Models
  - Multi-Criterion → lots of dimensions
    → weigh one vs. another



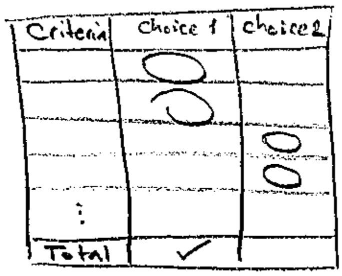

  - Probabilistic → uncertainty
    → balance risks vs. reward





# * Multi Criteria Decision Making

- Qualitative



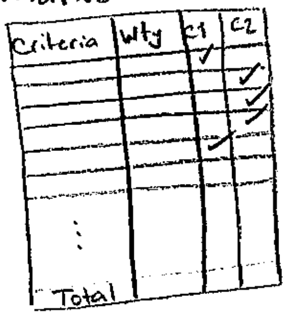

Recognize the dimension
weight the alternative along them
make choice

— If I make the decision but I don't like it

↳ Missing criteria ⟶ Add that criteria

↳ Weights ⟶ Quantitative

- Quantitative




↗ Normatively: why we like it? Understand ourselves

↘ Positively: why someone make that choice? Say something about his important criteria

— If the results are incosistent with qualitative ⟶ choice was quantitative and we know which criteria to suspect

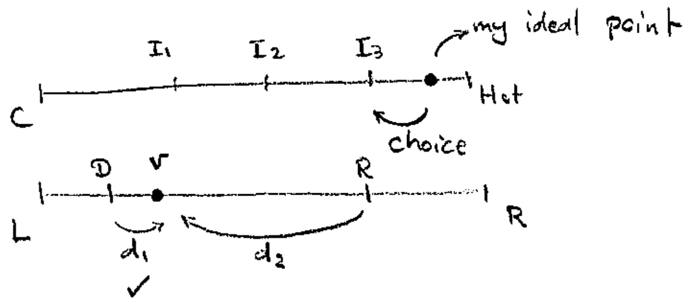
- Spatial Choice Models

— Thinking about ideal point

☑ Indian Food Hotness

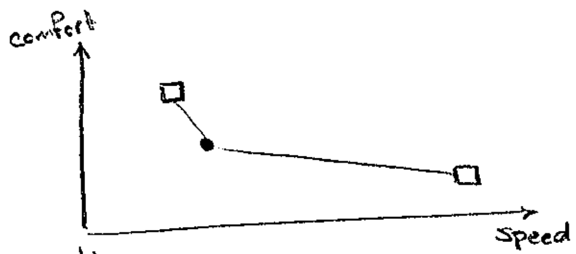
☑ Voting

↳ Hotelling's model




— Expanding to more dimensions

☑




— Normative

| Attr. | My ideal | Choice | \|diff\| | Choice 2 | \|diff\| |
|---|---|---|---|---|---|
| | KNOWN | KNOWN | | KNOWN | |
| | | total | | | |

⟹ Help us decide

— Positive

| Attr | Choice 1 | Choice 2 | Ideal |
|---|---|---|---|
| | 2 | 2 | ? |
| | 3 | 3 | |
| | 0 | 0 | |
| | ≥ | ≥ | |

⟹ we observed the choice (Revealed Preference)
Eliminate attributes with same values
Each of remaining attributes or a combination of them could be the ideal point

↳ voronoi diagram

## * Decision Making Under Uncertainty

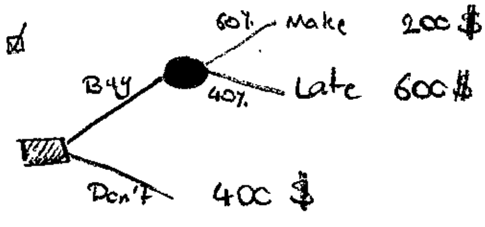
- Probability Axioms
  - → prob. of an outcome in [0,1]
  - → Sum of all possible outcomes equal to 1
  - → A ⊂ B ⟹ prob(B) ≥ prob(A)

- outcomes → all possible results
  events → subset of outcomes
  probability → the odds of something to happen

- probability
  - → Classical
  - → Frequency → estimate → counting
    - → stationarity assumption
  - → Subjective → use a model
    - → Prone to bias → should satisfy axioms especially axiom ③

- Decision Trees
  - Useful
    - → To infer odd things about the world → Positive
    - → Make better choices → Normative
    - → Learn about ourselves
  - Branches = choices → choose the branch with highest payoff

☑

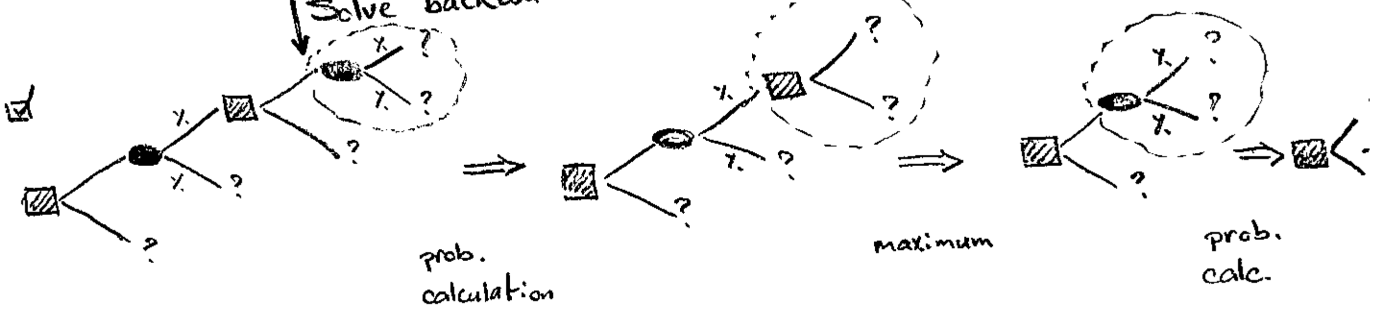



60% * 200 + 40% * 600 = 360 $

Buy

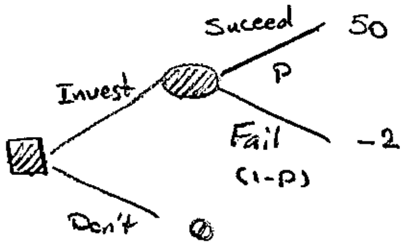
  - Procedure → Draw tree
    - Write down payoffs and probabilities
    - Solve backward

☑



- Infer probability
  ↳ learn prob. from others decision

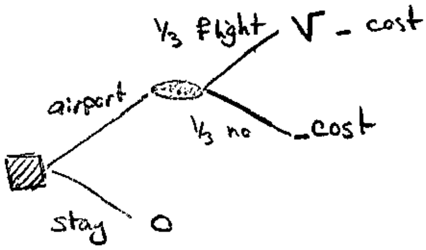
☑ Invest 200 \$ ⟶ Pay 50000 \$ ⟶ She's in!




she thinks:
$50P - 2(1-P) > 0 \longrightarrow P > \frac{2}{52} = 4\%$

- Inferring payoff
  ↳ learn about yourself

☑ ticket to visit parents ⟶ airline says 1/3 sucess chance ⟶ you don't go
how much did you want to meet the parents?!




cost: taxi - ...

$\frac{1}{3}(V-C) + \frac{2}{3}(-C) < 0 \Longrightarrow V < 3C$

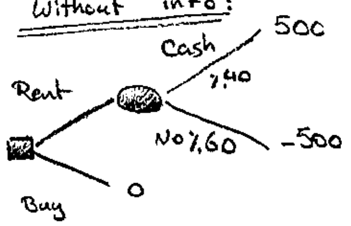
• Value of Information

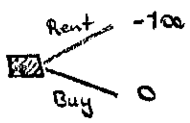
- Roulette wheel ⟶ value of information whether your number won = $\frac{\text{prize}}{38}$ (↗ I can tell you that before the wheel goes)
  38 slots ↘ value of information what is winning number = prize

- How to calculate? ⟶
  1. Calculate the value without the information
  2. Calculate the value with the information
  3. Calculate the difference

☑ Option A: buy car now
Option B: rent for 500\$ this month
40% chance that the car company offers 1000\$ cash back next month
what will it worth to you whether they have a cash back or not? How much would you pay him?

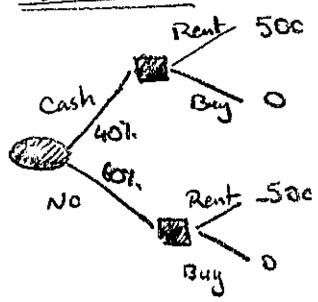
Without info:

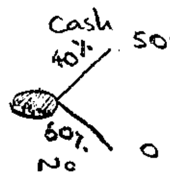



⟹ 


⟹ Buy 0 \$

With info:




⟹ 

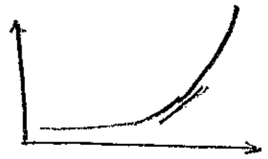

⟹ 200 \$

Value of Info = 200 - 0 = 200 \$

# * Tipping Points

- Small changes → big effect
  non-linear

- kinks & curves
  ↳ not exactly



→ not a tipping point, just exponential growth

tipping point: small change has large effect

- Physics: Percolation (تراوش) Model → can water find its way to system
  Epidimiology: SIS Model → Susceptiple > Infected > Susceptiple

- Types → Direct Tips: same variable tips the same entity
  ↘ Contextual Tips: sth changes in environment that make it possible for sth to happen and move from one step to another

Types → Tips between classes: stable ⇒ periodic, periodic ⇒ random
↘ Tips within class: equibilirium ⇒ another eq.

Malcom Gladwell
book: The Tipping Point

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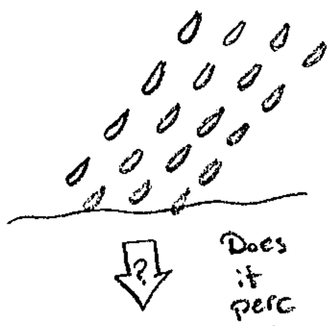
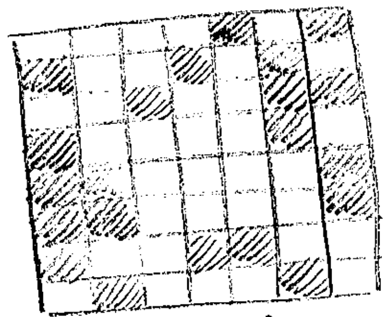
# * Perculation Model



Does it perc to the soil?

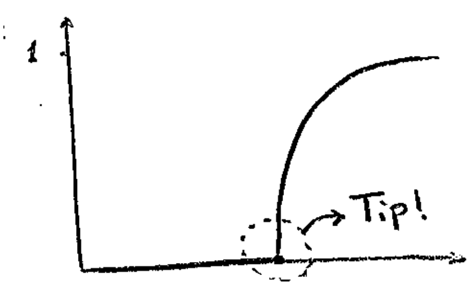
⇒



Jump only from filled in square to anothe

⇒ No way from top to bottom
= no perculate

p = prob. of a square to be filled
↳ Does it perculate?

- 

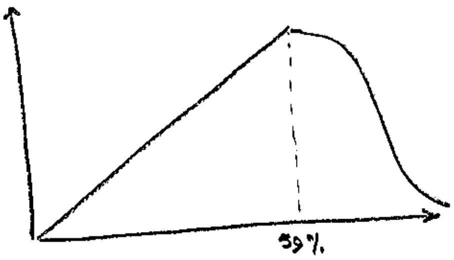

P↓ → lots of empty squares

p > 59% → so many squares filled in that its really likely to see perculation

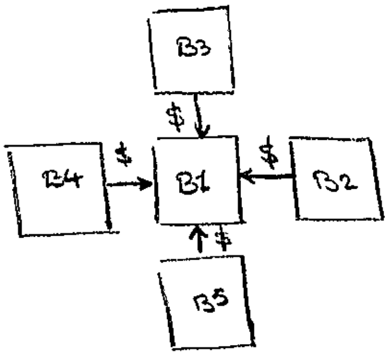
☑ Model fertility → Forrest Fires
NetLogo Software

59 % ← Fire ⇒ [Forrest Density] ⇒ Fire?
Tipping Point

☑ Yield Curve → wood from Forrest by Forrest Density




• Perculating Banks



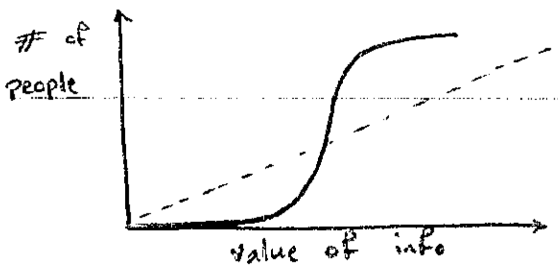

If B1 fails and borrowed a lot of money from other banks, it make them to fail too.

In modelling real banks, they add lots of details and they ask is there a tipping point?

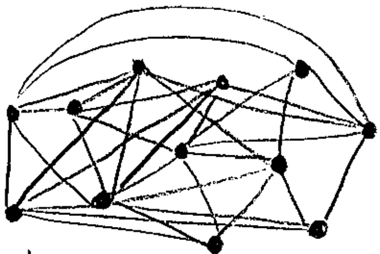
↳ More interconnected → failure spreads to more banks

• Information Perculation

What's the prob. that information perculates? → if rumor is juicy enough or info. is import. enough then it perculates




→ not linear : why? — because information spreads through network.

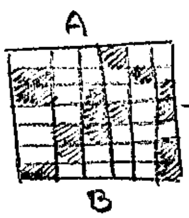


• Fertility : proof and innovations

— why after a long time ppl come to a breakthrough at the same time?

☑ Mathematical Proof

Steam Engine Innovation




→ information & knowledge filling more squares

↳ many paths become available suddenly

↳ burst of activity in area

* SIS Model

☑ Suceptible → Infected → Suceptible

↳ because the virus mutated (SIS)

Suceptible → Infected → Recover ⤳ never get that infection again (SIR)

• Basic Reproduction Number (R)

$R > 1$ → everyone gets that infection

$R < 1$ → noone get that infection

- Diffusion Model
  - Everybody gets it → noone cured
  - New disease called #

    $W_t$ = # of # at time $t$

    $(N - W_t)$ = # without

    $\tau$ = transmission rate
    (likelihood that someone get the disease from someone already have)

☑ information, ...

- Contact Rate: how often people meet
  ↳ c

  people may don't meet very often (c↓)

  people may meet a lot (c↑)

$$W_{t+1} = \underbrace{W_t}_{\text{already infected}} + \underbrace{(Nc)\tau \frac{W_t}{N}\left(\frac{N-W_t}{N}\right)}_{\text{get infected in this iteration}}$$

Spread:

Someone with # meet Someone w/o

↓ $W$      ↓ $N - W$

prob. of a meeting between two group

$$\frac{W}{N} \times \frac{N-W}{N}$$

↓ prob. of transmission

$$\tau\left(\frac{W}{N}\right)\left(\frac{N-W}{N}\right)$$

↓ prob. of meeting occurance

$$c \times \tau \times \left(\frac{W}{N}\right)\left(\frac{N-W}{N}\right)$$

↓ N people in population

$$(Nc)\tau\left(\frac{W}{N}\right)\left(\frac{N-W}{N}\right)$$


$W = 1\% \Rightarrow \frac{W}{N}\left(\frac{N-W}{N}\right) \sim \frac{1}{N}$ slow growth

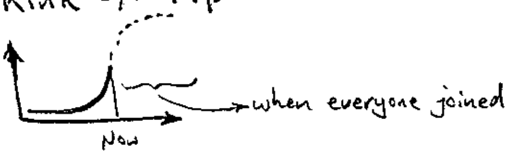
$W = \frac{N}{2} \Rightarrow \frac{W}{N}\left(\frac{N-W}{N}\right) \sim \frac{1}{4}$ fastest growth

$W \to N \Rightarrow \frac{W}{N}\left(\frac{N-W}{N}\right) \sim \frac{1}{N}$ slow growth

☑ In a school of 800 students, rumor spreads at a faster rate when 700 ppl have heard than when 50 have heard.

- No tipping points → only accelration ⇒ Kink ≠ Tip

☑ Facebook growth is a pure diffusion model




- SIS Model
  - After infection people can get back to susceptible pool again.

$$W_{t+1} = W_t + \underbrace{(Nc)\tau \frac{W_t}{N}\left(\frac{N-W_t}{N}\right)}_{\text{new}} - \underbrace{aW_t}_{\text{cured}} = W_t + W_t\left(c\tau \frac{N-W_t}{N} - a\right)$$

- $W_t$ ~ small $\longrightarrow \frac{N-W_t}{N} \sim 1 \longrightarrow W_{t+1} = W_t + W_t(c\tau - a)$

  ↳ $c\tau - a \overset{?}{>} 0 \longrightarrow c\tau \overset{?}{>} a \longrightarrow \underbrace{\frac{c\tau}{a}}_{R_0} \overset{?}{>} 1$

  $R_0 \longrightarrow$ basic reproduction number $= R_0 = \frac{c\tau}{a}$

$R_0 > 1$ : spreads
$R_0 < 1$ : dies of
$\Rightarrow$ 1 is tipl

☑ Measles ≈ 15 سرخک → SIR
Mumps ≈ 5 کشک → SIR
Flu ≈ 3 آنفلانزا → SIS

- Models ⟶ design policies ⟶ Vaccines!

$V$ = % of vaccinated

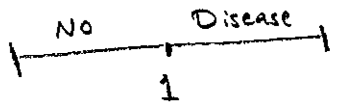
$r_0 = R_0(1-V) \leqslant 1 \longrightarrow V \geqslant 1 - \frac{1}{R_0}$

☑ Measles ⟶ $\frac{14}{15}$ to stop spreading
Mumps ⟶ $\frac{4}{5}$

We have tipping points here too! ⟶ if we vaccinated 75% of people but we should vaccinated 80% it not gonna work!!!

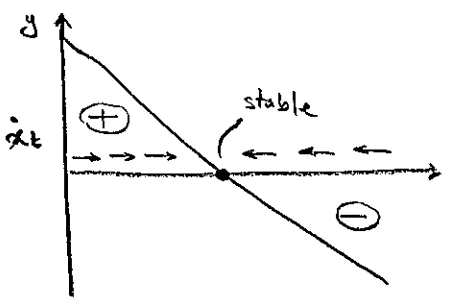
- Diffusion vs SIS

no tip | $R_0$ tipping Point ⟶ vaccine reduces $R_0$

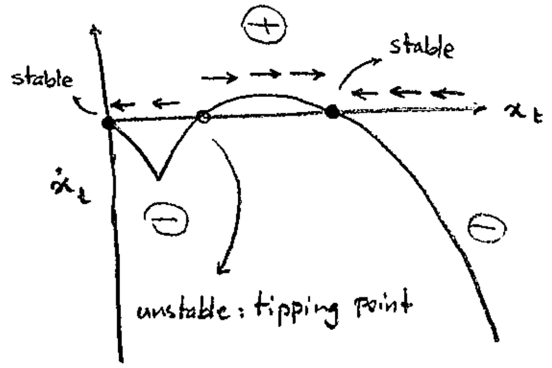



## * Formal Forms of Tipping Point

- Dynamical System




Equilibrium




point + ε ⟶ move right
point − ε ⟶ move left

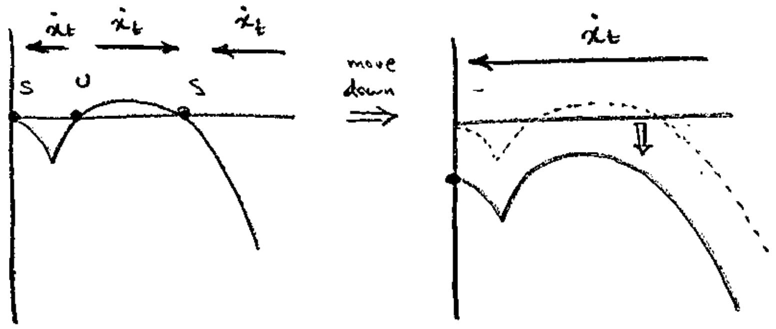
* any slight movement will lead to a large change in variable ↲

Direct Tip

- Direct Tip: small action or event that has a large effect on end state

☑ In World War II → killing of Archduke Ferdinand → Tip whole world to war
tens of millions people die

– To prevent tips we change the contex




2 stable, 1 unstable ⟼ .. equilibrium

← → ← ⟼ always move left

Contextual Tip: change in the environment by a tiny bit has a huge effect on the end state (percolation model)

☑


percolation model
↓
contextual tip

☑ SIS Model

Basic Reproduction Number

$R_0 > 1 \Rightarrow$ spread

$R_0 = \frac{cT}{a}$ → changing the context

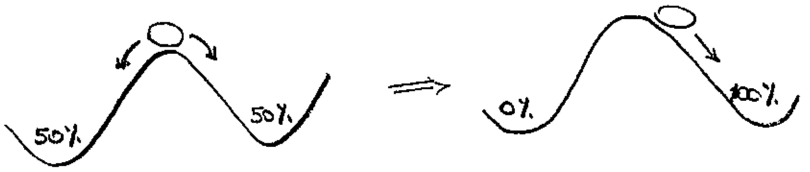
- Between and within Class
  - Classes: equilibrium, periodic, random, complex
  - between class → change from one class to another
    - ↳ by changing variable system goes from eq. to periodic to complex...

☑ tempreture goes up ⇒ bees awaken and scurry randomly about to cool hive

↳ contextual, between classes tipping point

# * Measuring Tips → how tippy a system is?

- Idea of Measuring




⟹ To measure tipping points → Measure changes to outcomes

Active Tip
probable outcomes

Tipped
We know what happened
Uncertainty goes away

- Diversity Index

| Possible outcome | Prob |
|---|---|
| A | 1/4 |
| B | 1/4 |
| C | 1/4 |
| D | 1/4 |

$P_A + P_B + P_C + P_D = 1$

diversity $(1/4, 1/4, 1/4, 1/4) >$ diversity $(1/2, 0, 1/2, 0)$

– first compute the prob. of if two ppl. meet, they are from the same type:

$$P_A \cdot P_A + P_B \cdot P_B + P_C \cdot P_C + P_D \cdot P_D = \sum_{i=A,B,C,D} P_i^2$$

$P_A = P_B = P_C = P_D = 1/4 \longrightarrow \sum P_i^2 = \frac{1}{4}$

second, reverse the number: 4 → 4 types here

– Diversity Index $= \frac{1}{\sum P_i^2}$

☑ $P_A = \frac{1}{2}$ $P_B = \frac{1}{3}$ $P_C = \frac{1}{6}$ $\longrightarrow$ D.I $= \frac{36}{14} = 2\frac{4}{7} \leq 3$

approximately tells us how many things we have

we can say that initially there are $2\frac{4}{7}$ places to go, but after tipping happe[ns]

there is one place (e.g. A) to go.

☑ $P_A = \frac{1}{2}$ $P_B = P_C = P_D = \frac{1}{6}$ $\longrightarrow$ D.I $= 3 \leq 4$

- Entropy Index

$$H = -\sum_i P_i \log_2(P_i)$$

☑ $P_A = P_B = P_C = P_D = \frac{1}{4}$ $H = -\sum \frac{1}{4}\log_2(\frac{1}{4}) = 2$

– Entropy tells us the number of bits of information we need to know the outcome.

☑ Possible outcomes

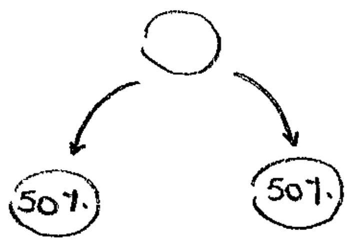
A, B, C, D $\xrightarrow{Q_1: (A,B) \text{ or } (C,D)}$ A, B $\xrightarrow{Q_2: A \text{ or } B}$ B

- Diversity Index : # of types
  Entropy : amount of information

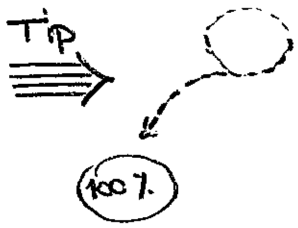
☑ $P_A = P_B = P_C = P_D = \frac{1}{4}$

D.I. = 4 → 4 items and equally likely
Ent = 2 → with 2 questions we can say which type we

☑




D.I. : 2
Ent : 1



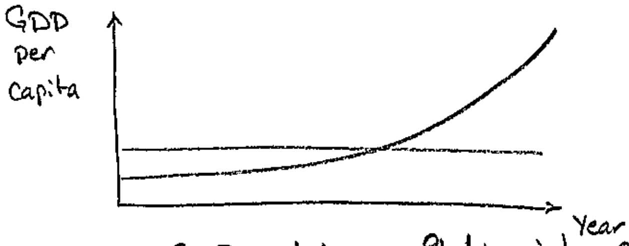

D.I: 1
Ent : 0

How much system tipped → DI: 2 - 1
→ Ent: 1 - 0

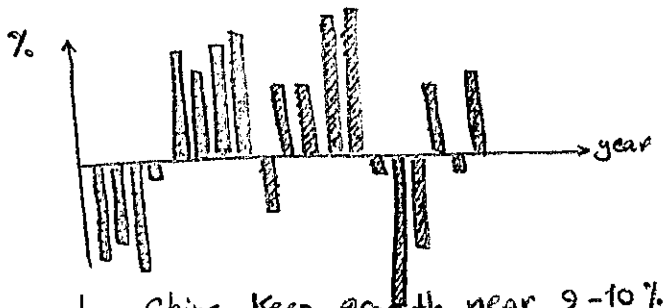
- Tips may increase indexes (eg. equilibrium ⇒ random) or decrease it.
- Tips : Change in likelihood of outcomes not Kink
  ↳ sth happen that we don't expect
  sth that we expect won't happen

# * Economic Growth

- Definitions
  - GDP = Gross Domestic Product : Total market value of all goods and services

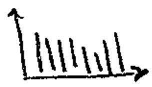


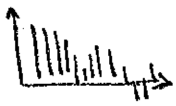

  - Real GDP : taken inflation into account




annual chang in Real GDP

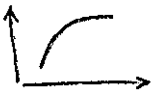
  - why China keep growth near 9-10% in last decade?
    why Japan lose that growth?



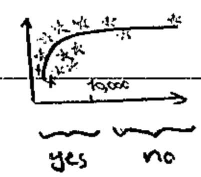


  - Put money in bank → exponential growth

  - Put money in machinery → they get old



  - Life satisfactory vs GDP per Capita
    - ↳ does money make you happier?
      - ↳ basic services : medical care, education, ...




- Exponential Growth
  - X : dollar → put in bank; r% interest → X(1+r) → each year
  - ☑ 100 \$ 5%    100\$ → 105 \$ → 110.25 \$

    10 years : $100 * (1+0.05)^{10}$
  - G : GDP per Capita → r : growth rate → 10 years : $G(1+r)^{10}$

| Year | 2% | 6% |
|---|---|---|
| 0 | 1000 | 1000 |
| 1 | 1020 | 1060 |
| 10 | 1219 | 1791 |
| 35 | 2000 | 7686 |
| 100 | 7245 | 339,302 |

  - Rule of 72 : Divide the growth rate into 72 and it tells you approximately time for GDP to DOUBLE
  - ☑ $r = 2\% \longrightarrow \frac{72}{2} = 36$ years

    $r = 6\% \longrightarrow \frac{72}{6} = 12$ years

$d = 0.25$ , $S = 0.3$ , we startout with 4 machines

Year 1 :

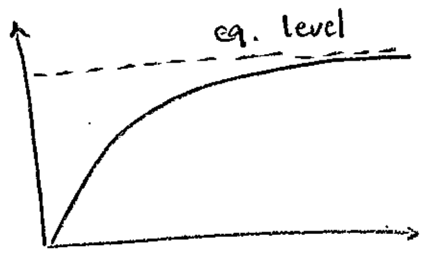
| | | |
|---|---|---|
| $M_1 = 4$ | ▷ $M_2 = 9$ | ▷ $M_3 = 16$ |
| $O_1 = 10\sqrt{4} = 20$ | $O_2 = 10\sqrt{9} = 30$ | $O_3 = 10\sqrt{16} = 40$ |
| $I_1 = 20\,(0.3) = 6$ | $I_2 = 30\,(0.3) = 9$ | $I_3 = \dots$ |
| $Dep_1 = 4\,(0.25) = 1$ | $Dep_2 = 9\,(0.25) \approx 2$ | |

⟹ Contineous Growth — Will it stop growing?

400 Machine

| | |
|---|---|
| $M_1 = 400$ | ▷ $M_2 = 360$ |
| $O_1 = 10\sqrt{400} = 200$ | |
| $I_1 = 200\,(0.3) = 60$ | |
| $Dep_1 = 400\,(\frac{1}{4}) = 100$ | |

⟹ Reduced — where is the stop point ?
↳ The Equilibrium




- Long Run Equilibrium:

Investment = Depreciation

Investment $= 0.3 \times 10\sqrt{M} = 3\sqrt{M}$

Depreciation $= M/4$

$3\sqrt{M} = \frac{M}{4} \Rightarrow M = 144$ → $I_t = Dep_t = 36$
↘ $O_t = 120$

{ Depreciation : Linear
{ Output : concave
⟹ They meet ⟹ Stop growing ⟹ The irony of growth model

- How do we get more growth?
  ↳ Innovation → Technological growth parameter

• Solow Growth Model

- Add one variable → Innovation → solve the problem

$L_t$ = Labor @t

$K_t$ = Capital @t

$A_t$ = Technology @t

$O_t$ = Output @ t

$$O_t = A_t K_t^{\beta} L_t^{1-\beta}$$

$\beta$ : different manufacturing processes
→ $\beta\uparrow$ capital intensive
→ $\beta\downarrow$ Labor intensive
→ $\beta = \frac{1}{2}$ like previous model

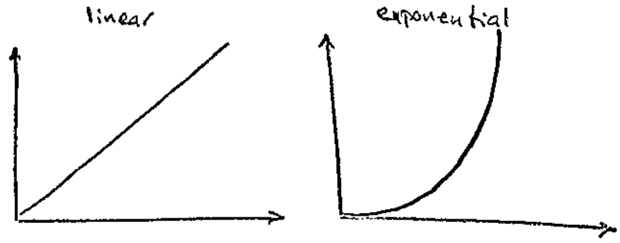
- Continuous Compounding

Annually : $G(1+r)^t$

Daily : $G\left(1+\frac{r}{365}\right)^{365t}$

Hourly : $G\left(1+\frac{r}{365 \times 24}\right)^{365 \times 24\, t}$

Limit : $\lim_{n \to \infty} \left(1+\frac{r}{n}\right)^{nt} = e^{rt}$ $(e = 2.71828)$ $\Rightarrow$ exponential




- Simple Growth Model

  - workers
    Coconuts
    Picking Machines
    Machines wear out / depreciate

$L_t$ = workers @ t

$M_t$ = Machines @ t

$O_t$ = Output of coconuts @ t

$E_t$ = Number consumes @ t

$I_t$ = Number invested @ t

$s$ = Saving rate

$d$ = Depreciation rate

Assumption 1 : Output is increasing and concave in Labor and machines

Machine ↑ ⇒ coco ↑

Workers ↑ ⇒ coco ↑

$$O_t = \sqrt{L_t M_t}$$

Assumption 2 : Output is consumed or invested

No waste!

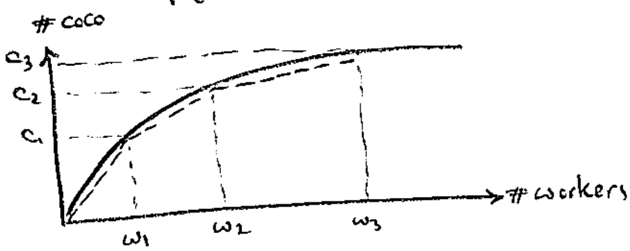
$$O_t = E_t + I_t$$

$$I_t = s \times O_t$$

Assumption 3 : Machines can be built but depreciate

$$M_{t+1} = M_t + I_t - d M_t$$

- Concave → the first machine is worth more than the second, the second machine is worth more than the third ... → deminishing return to scale




☑ Simplification : $L_t = 100$

$O_t = \sqrt{L_t}\sqrt{M_t} = 10\sqrt{M_t}$

Realistic Model: workers depends on the wages, wages function of output depends how much they like coconuts ...

☑ $O_t = 10\sqrt{M}$

$I_t = 0.3 \times 10\sqrt{M} = 3\sqrt{M}$

$Dep_t = 0.25M$

Eq: $0.25M = 3\sqrt{M} \rightarrow M = 144$

$O_t = 120$

$\xrightarrow{A_t = 2}$ innovation that make coconut machine twice as productive

$O_t = 20\sqrt{M}$

$I_t = 6\sqrt{M}$

$Dep_t = 0.25$

Eq: $M = 496$

$O_t = 480$

- technology changes : ① get more productive
  ② getting more stuff ⟹ more investment ⟹ 4x Output

INNOVATION MULTIPLYER

↳ Innovation multiplyer → labor and capital more productive
↘ incentives to invest in more capital

- productivity 2x

long run GDP 4x

Puzzle: Is it additive or multiplicative? ( $2+2=4$ ? $2^2=4$ ? ) → Model

☑ $A_t = 3$

$O_t = 3 \times 10\sqrt{M}$

$I_t = 0.3 \times 30\sqrt{M} = 9\sqrt{M}$

$Dep_t = M/4$

Eq: $9\sqrt{M} = \frac{M}{4} \longrightarrow M_t = 36^2$

$O_t = 3 \times 10\sqrt{36^2} = 1080$

(9X) $O_t = 120$

↳ Multiplicative

↳ proof :

$O_1 = \sqrt{L}\sqrt{M}$

$I = s\sqrt{L}\sqrt{M}$

$Dep = dM$

Eq: $dM = s\sqrt{L}\sqrt{M}$

$\rightarrow \sqrt{M} = \frac{s}{d}\sqrt{L}$

$\boxed{O_1 = \frac{s}{d}L}$

$O_2 = a\sqrt{L}\sqrt{M}$

$I = sa\sqrt{L}\sqrt{M}$

$Dep = dM$

Eq: $dM = sa\sqrt{L}\sqrt{M}$

$\rightarrow \sqrt{M} = \frac{sa}{d}\sqrt{L}$

$\boxed{O_2 = a^2\frac{S}{d}L = a^2 O_1}$

- Simple Growth Model → growth stop → GDP depends on saving rate

Solow Growth Model → growth can continue with changes to technology (A ↑)

Where A comes from? Labor!

- Endogenous Growth
  - Labor can go to output or technology / idea creation → investing in new tech
    ↘ R&D
  - growth ceases w/o innovation → first invest in capital, then invest in technology
  - Choice : how much people go to pick coconuts? how much go to tech?

* Can China continue its growth?



→ GDP per Capita

$d = 0.1$
$S = 0.2$
$M = 3600$
$L = 10000$

$O = 100\sqrt{3600} = 6000$
$I = 0.2 \times 6000 = 1200$
$D = 0.1 \times 3600 = 840$
$M = 4440$

▷ $O = 100\sqrt{4440} = 6700$
$I = 0.2 \times 6700 = 1340$
$D = 0.1 \times 4400 = 440$
$M = 5340$
Growth = 11 %

▷ $O = 7300$
$I = 1460$
$D = 530$
$M = 6270$
Growth = 8~9 %

$M = 10{,}000$ ...

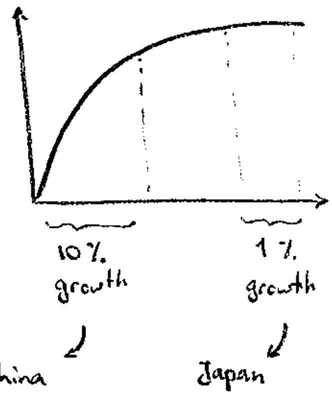
⇨

$O = 10000$
$I = 2000$
$D = 1000$
$M = 11000$
$O^{+} = 10500$
$G = 5\%$

⇨

$M = 22{,}500$ ...

$O = 15000$
$I = 3000$
$D = 2250$
$M = 23250$
$O^{+} = 15250$
$G = 1\sim2\%$




Once the capital of people gets high enough the growth rate falls.


Growth → Capital Accumulation
Growth → Technological Advancement

* Why some countries are poor?

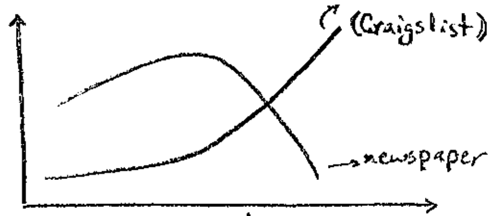
- Model $Q = AK^{\beta}L^{1-\beta}$ → Tech, Capital, Labor

Book: Why Nations Fail?

- Things not in model → Inequality, Culture
- Growth require a strong central gov. to protect capital & investment but that gov. cannot be controlled by a select few.
  - investment ↳ in machines → theft security
  - ↳ in innovation → copyright
  - ↳ extraction : limits growth by lowering investment in innovation & capital
    - ☑ bribe, corruption, money storage → extract money → hurts economy by multiplier effect
- A ↑ ⟹ less labor required for same output → unemployment
  - ↳ so growth requires creative destruction → sth new destroys older ones!

☑ Movies


☑ Ads




250K jobs lost in newspapers.
23 jobs created in Craigslist !!!

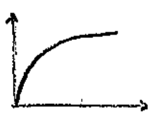
↳ powerful gov controlled by few → tends to save jobs → lower productivity
→ may oppose the new technology → gov. controlled by old industry

- Model fertility
  - increase personal GDP
    - ↳ A↑ : successful ppl continue to learn
    - ↳ L↑ : work harder

# ✱ Model of People

- "Imagine how hard physics would be if electrons could think!"
  — Mary Gelmont
- Difference between people & electrons
  - purposeful Actors → Goals
    → Belief Structure
    → Messy
    ⇒ How we are gonna behave?!
  - Diverse → Different goals and objectives
- Framework
  - Rational Actor Model → people optimize
    → unrealistic
    → good benchmark
  - Behavioral Model → lots of data of how real people actually make decision
    → build a model as close as possible to "how ppl behave"
    → too complicated
      ↳ simplification ⇒ rationality + biases that ppl may ha
  - Rule Based Model → assume ppl follow rules
- Rational
  - Objective → mathematical function
  - Optimize
  - ☑ How many hours should a guy work per day?
    - C = consumption
    - L = leisure

    Utility = $\sqrt{C}\sqrt{L}$ → square root function, deminishing return ?



- Behavioral
  - Observe → people are not rational
    ↳ but they are not rational is systematic ways
    → Psychology
  - Neuro science → how ppl think because of their brain shape
    → why ppl think irrational

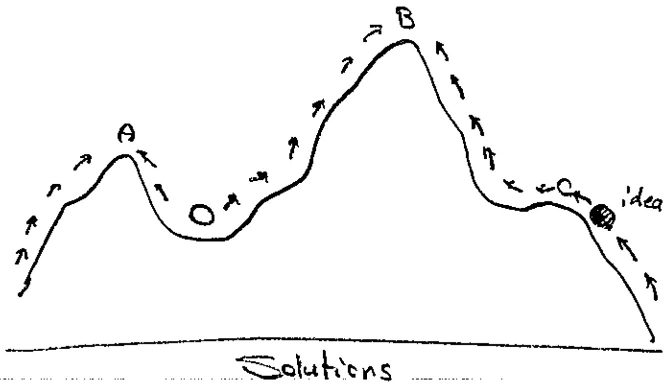
- Rule - based → simple rules ☑ Schelling Model
  ↘ if rule is close to what ppl do it is sufficient

# * Problem Solving:

- Problem → action a
  ↘ function F → value of action F(a)
  Pay off function

☑ a: code → F(a): execution speed
a: healthcare policy → F(a): efficiency of that policy

- a is solution → F(a) is how good that solution is
- Metaphore of a landscape
  ↳ altitude: value
  ↳ B: best solution




- improve an idea
  ↳ climb uphill

---

- C & A → local best

- Perspectives: how you present the problem
  - how you encode the problem
  - different perspectives → different landscapes

---

- Heuristics: how you move on landscape
  - hill climbing: climbing up the hill
  - random search: randomly pick points and compare values
- Problem solving of people
  ↳ Perspectives + Heuristics

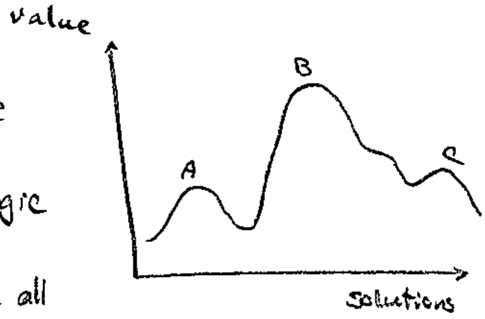
- Teams
  - groups of people → people are different

- Recombination
  - I have some solution for one problem
    U have a solution for a different problem
    Combine your solution with mine → better solution
  - Sophisticated products = $\Sigma$ solutions to sub-problems
- Growth depends on sustained innovation

Diversity $\xrightarrow{\Sigma}$ Innovation $\xrightarrow{\Sigma}$ better innovation

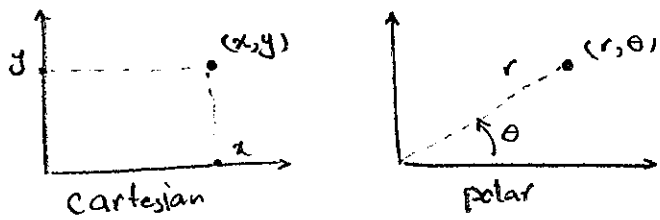
## * Perspective

- how you represent a problem
- solving problem → finding high points in the landscape
  - ↳ formalize landscape → better logic




- Formally, a perspective is a representation of the set of all possible solutions.
  - provides encoding → assign a value to each encoding → landscape

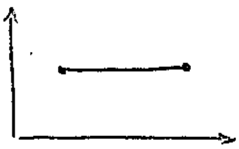
☑ plot points




- which one is better?

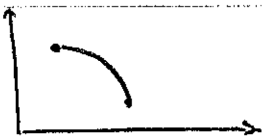
it depends!

↳ describe a line



cartesian : easy ✓
polar : phew!

↳ describe an arc



cartesian : hard
polar : easy ✓

perspective depend on the problem

☑ Mandaleev Table → atomic weight → structure → e.g. column of metals ✓
↘ alphabetical order

There were gaps in his representation → Sc (discoverd 1879)
Ga ( " 1875 )
Ge ( " 1886 )

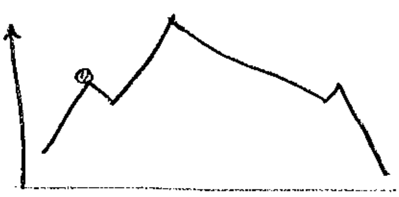
atomic weight perspective → have holes → useful → guide ppl to discover

☑ Hiring someone → organizing lots of applicants

Value Competence : GPA
Value Work Ethic : thickness of CV!
Value Creativity : colorfulness of CV!

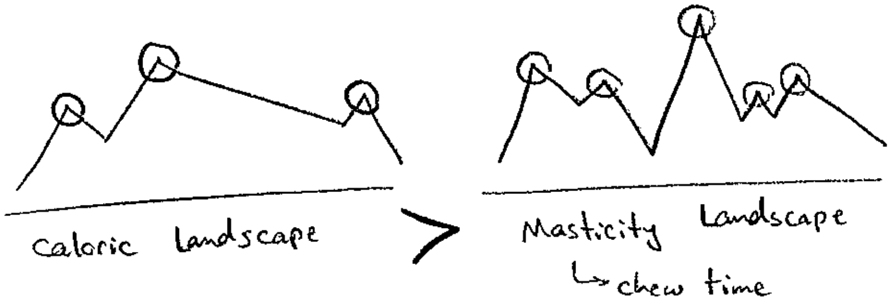
⟹ different ways to organize applicants depending on job

- "Rugged Landscape"
  - lots of peaks

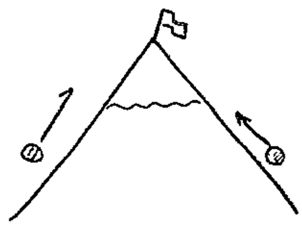


- Local Optima : an action a, such that neighboring actions have lower values
  - a peak on the landscape
  - better perspective have fewer local optima!
  - bad perspective have a lot of them

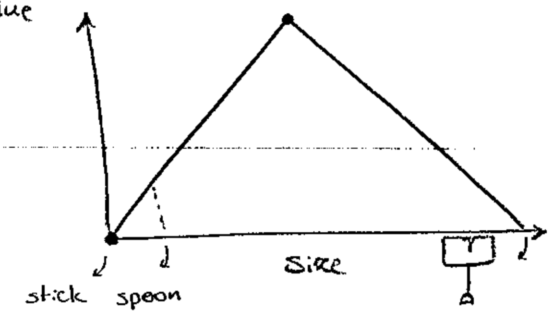
☑ Best candy bar




- best (ideal landscape)
  - ↳ one peak
  - ↳ called "Mt Fuji Landscape"
    - ↳ one giant cone
  - ↳ easy to find way to the top



☑ Optimal size of the shovel
- ↳ by Fredrick Taylor
- ↳ value : how useful the shovel is at the task
  - ↳ how many pounds of coal mined
- ↳ idea of drawing landscapes and finding our way to the solution in the top ⇒ scientific mgmt.



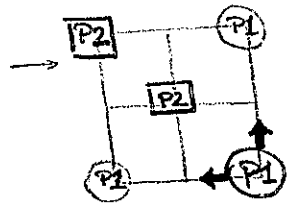

☑ Sum to Fifteen Game
- ↳ by Herb Simon — Noble prize winner in economics
- ↳ to show ppl why different perspective is useful.
- ↳ Game : cards numbered 1∼9 face up on table
  players alternate selecting cards
  win if you hold exactly three cards that sum to fifteen
- ↳ scenario 1 :
  P1 : 4 - 6 - 2 → picking six looks strange but tricky!
  P2 : 5 - 8 - lose anyway!

  1 - 3 - 7 - 9

- ↳ A magic Square

| | | |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

→ every row, col, or diag adds up to 15!

→ 

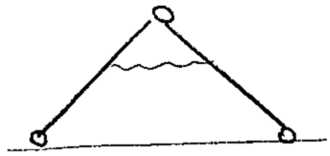

→ scenario 1 in the form of tic-tac-toe!

- Savant Existance Theorem: For any problem there exists a perspective that creates a Mt Fuji landscape.

  ↳ in fact many do.

  ↳ intuition: assume we have all solutions
  put best one in middle
  put worst ones on both ends
  line up solution to form Mt. Fuji!

  ↳ not a good way to solve problems
  ↳ it exists



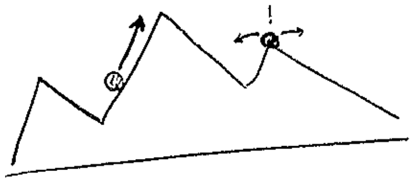
- Bad Perspectives: With N alternatives there are N! ways to create 1D landscapes

  ↳ most of those are not very good ⟶ can't organize in useful way.
  ↳ a few good that are close to Mt. Fuji ⟶ Takes a genious to find it ⟶ Newton!

✳ Heuristics: how you go about finding solutions to problems once you represented them.

- Hill climb: move locally to better points

  ↳ stuck at local optima
  ↳ one of many solutions




- Do the opposite: think of an existing solution ⟶ do the exact opposite!

  ☑ you go buy sth ⟶ somebody tell you the price
  solution: you tell the price! ⟶ company, airline, hotel ...

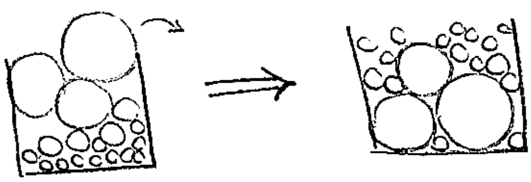
  ☑ companies try to lower the price
  Solution: set a higher price ⟶ signal the quality

  ☑ grilled cheese sandwich ⟶ cheese between bread!
  solution: put the cheese outside the bread!

- Big Rocks First: deal with important things first

  ↳ Stephan Covey ⟶ what makes ppl successful ⟶ books full of heuristics

  ↳ small rocks first?

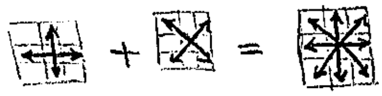


- Wolpert & McCready: No Free Lunch Theory: All algoriths that search the same number of points with the goal of locating the max value of a function defined on a finite set, perform exactly the same when averaged over all possible functions.

  – two heuristic, search the same amount of solutions (e.g. random search vs. do the opposite) for all possible problems (some hard, some easy) ⟶ no heuristic is any better than any other

– Informal : Unless you know sth about the problem being solved, no algorithm or heuristic performs better than any other

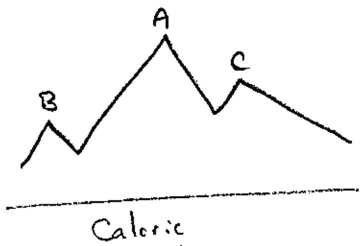
- Diverse Heuristics
  - combine two heuristics → search more points

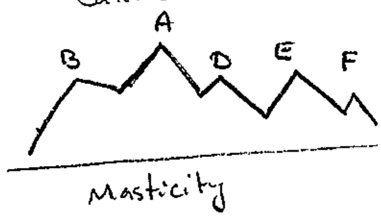
☑ 

# * Teams

- people working together and brainstorming ideas
  people collaborating over time and improving ideas

- Different Perspectives
  - global optima ramains the same in all landscapes
  - local problem solvers characterized by their local peaks
    ↳ fewer peak ⇒ better perspective
  - assign a value to each of peaks
    ↳ average of their values over different perspective




⇓

Caloric : A , B , C

Masticity : A , B , D , E , F

⇓

| Peak | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Value | 10 | 8 | 6 | 6 | 2 | 4 |

- what is the average value of peak for each problem-solver
  ↳ call it «Ability»
  ↳ higher ability ⇒ better problem solver

- work in teams

☑ scenario 1

Caloric : A , (B) , C

Masticity : A , (B) , D , E , F

⤵ B

↳ They both stuck at B

☑ scenario 2

Caloric : A , B , (C)

⤵ C

Masticity : (A) , (B) D , E , F ⟶ C is not found, start to hill climb from C and find the best local optima after that → improve!

– Stuck Point : Intersection of local optima of problem solver ☑ A, B

Local optima for the team is the local optima for the individuals → Intersection property

☑ Cal : A , B , C / Mas : A , B , D , (E) , F ⤴ ⇒ Cal : A , B , (C) / Mas : A , B , D , E , F ⤵ ⇒ Cal : A , B , C / Mas : A (B) , D , E , F ⇒ <u>stuck in B</u>

- The ability of team is higher than ability of individuals

☑ Caloric

| A | B | C |
|---|---|---|
| 10 | 8 | 6 |

Ability : 8

Nasticity

| A | B | D | E | F |
|---|---|---|---|---|
| 10 | 8 | 6 | 4 | 2 |

Ability : 6

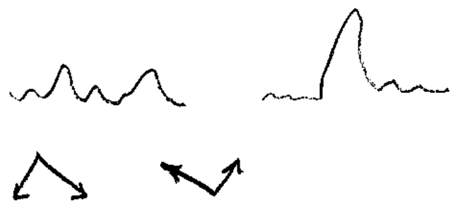
⟹ Team

| A | B |
|---|---|
| 10 | 8 |

Ability : 9

— teams are innovative → products get better

- The team can only get stuck on a solution that's a local optimum for every team member
  - We want people with different local optima
    - ↳ different perspectives → landscape
    - ↳ different heuristics → search strategy



- What's Missing ??

① Communication → misunderstanding
↘ not listening

↳ making an artifact (prototype) can be solution. (Design Thinking)

② Error in interpreting the value of solution → underestimating
↘ overestimating

↳ oracles → not always available

- Innovation comes from diversity → different ways of seeing problem
↘ different ways of finding solutions

## * Recombination

- Few ideas —recombination→ more of thes sort of ideas → more & more innovation

☑ 1 2 3 5 _ 13 → answer : 8
solution : add two left , subtract two right

1 4 _ 16 25 36 → answer : 9
solution : squares

1 2 6 _ 1806 → hard !

1 2 6 42 1806

1, 4, 36, 1764 → trick 1
$1^2$, $2^2$, $6^2$, $42^2$ → trick 2
} combined !

- You can recombine
  - ↳ Drives economy
  - ↳ Drives science → combine solutions ⟹ Geometric Explosion in number of Solutions

☑ How many ways to pick three objects?

$$\frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120 \longrightarrow 10 \text{ technologies} \sim 120 \text{ solutions}$$

☑ 52 cards in a deck, pick 20 of them

$$\binom{52}{20} \simeq 125 \times 10^{12}$$

☑ Economic Growth Model $\uparrow (A)\, K^{\beta} L^{1-\beta} \Rightarrow \text{growth} \uparrow$

(Book: Recombinant Growth by: Martin Weitzman)

- Exaptation
  - From biology ☑ birds develope feathers to keep them warm, but these feathers allowed them to fly
    - ↳ a feature having a function for which it was not originally adapted / selected
      - ↳ preadaptation : تطبیق سبق
  - you come with some innovation / solution for some reason → it get exapted → get used in another context
  - ☑ Laser ⟶ laser printers, laser pointers
  - something that is useless in a context maybe great in another → perspectives exaptation

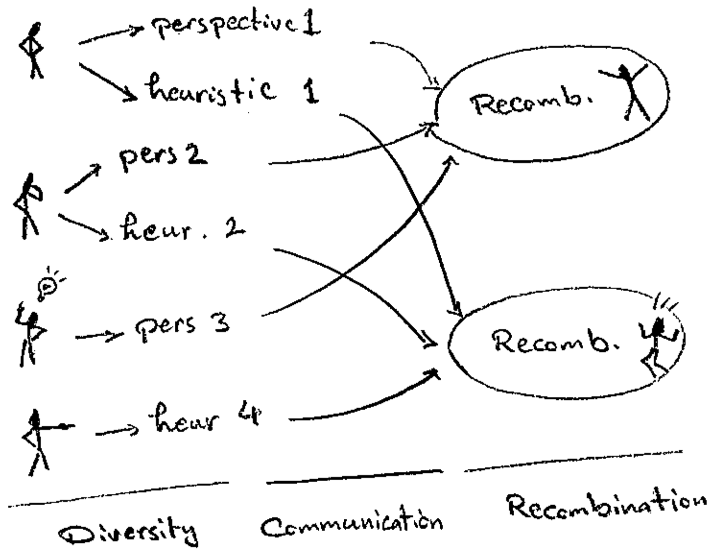
---

☑ Mastisity ⟶ bad for chocolate bars ⟶ failed solution
⟶ good for cereals

- Communication
  - some way to communicate ideas

(Book: Gifts of Athena by: Joel Mokyr)
↳ rise of modern university
↳ printing press
↳ scientific communication

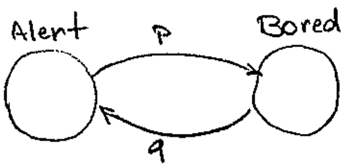
* Summary



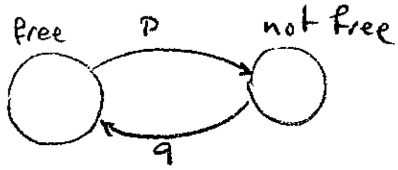
# * Markov Models 8

- Idea
  - ↳ states : person's psyche could be in, state of gov., economy situation
  - ↳ Transition Probability : prob. of moving from one state to another

☑ student




☑ country freedom





looks linear trend toward freedom

- Four class of behaviors for models.

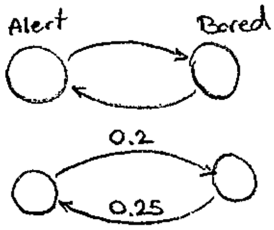
  Equilibria , Cycle , Random , Complex
  - ↳ Markov Assumption is here if a few assumption holds

- Markov Convergence Theorem
  - Assumptions : Finite # of states
    - probabilities stays fixed
    - Ability to get from one state to any other
  - we have equilibrium given these mild assumption

- why we learn MP?
  - Exaptation & Fertility
  - Always go to equilibria

- Dynamics

☑




20% of Alerts → Bored

25% of Bored → Alert

scenario #1 : 100 Alert → A : 100 → 80 → 64 + 5 = 69 → ...

B : 0 → 20 → 16 + 15 = 31

hairy calculation

- Markov Transition Matrix

| | $A_t$ | $B_t$ |
|---|---|---|
| $A_{t+1}$ | 0.8 | 0.25 |
| $B_{t+1}$ | 0.2 | 0.75 |

$$\rightarrow \begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \rightarrow \begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix} \times \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.69 \\ 0.31 \end{bmatrix} \rightarrow \cdots \begin{bmatrix} 0.58 \\ 0.42 \end{bmatrix}$$

after six periods 58% of students were alert

Scenario # 2 : 100 Bored

$$\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} \rightarrow \begin{bmatrix} \quad \end{bmatrix} \times \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \rightarrow \begin{bmatrix} 0.53 \\ 0.47 \end{bmatrix}$$

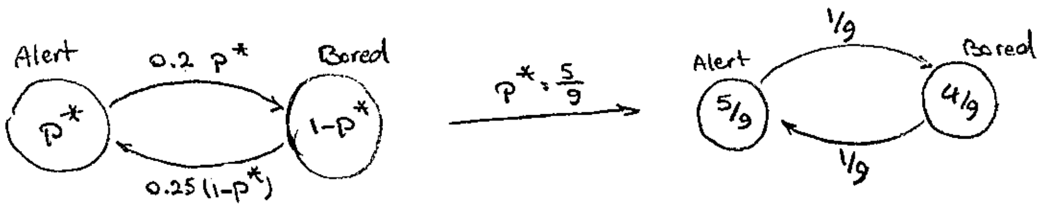
looks like near scenario # 1

- How to find equilibria ?

$$\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix} \times \begin{bmatrix} p^* \\ (1-p^*) \end{bmatrix} = \begin{bmatrix} p^* \\ (1-p^*) \end{bmatrix}$$

Means that after system goes to eq. point $p^*$ more iterations won't change $p^*$.

$$0.8\,p^* + 0.25\,(1-p^*) = p^* \longrightarrow p^* = \frac{5}{9}$$




- Equilibrium Point : nothing change

Statistical Equilibrium : World keeps churning but the distribution of types stays the same.

↳ people moving, dist don't change

☑ Democracy Model

Each decade 5% : democracy → dictatorship

20% : dictatorship → democracy

$$\begin{bmatrix} 0.95 & 0.2 \\ 0.05 & 0.8 \end{bmatrix} \times \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 0.425 \\ 0.575 \end{bmatrix} \rightarrow \begin{bmatrix} 0.52 \\ 0.48 \end{bmatrix} \rightarrow \text{Trend : look like linear !}$$

$$\begin{bmatrix} 0.95 & 0.2 \\ 0.05 & 0.8 \end{bmatrix} \times \begin{bmatrix} p \\ 1-p \end{bmatrix} = \begin{bmatrix} p \\ 1-p \end{bmatrix} \xrightarrow{p = \frac{4}{5}} \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$$

→ not every country going to have democra[cy]

counter intuitive

☑ Country Freedom

Each decade 5% of free & 15% of not free → partly free

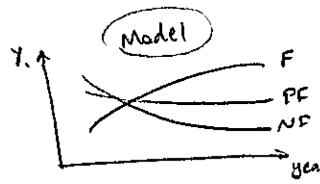
5% of not free & 10% of partly free → free

10% of partly free → not free

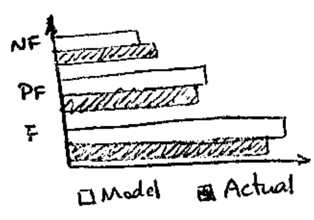
| | F | PF | NF |
|---|---|---|---|
| (new) F | 0.9 | 0.1 | 0.5 |
| (new) PF | .05 | 0.8 | .15 |
| (new) NF | 0 | 0.1 | 0.8 |

Next state ← Current state

$$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} F \\ PF \\ NF \end{bmatrix} = \begin{bmatrix} \text{New } F \\ \text{New } PF \\ \text{New } NF \end{bmatrix}$$




$$\begin{bmatrix} 0.9 & 0.1 & 0.5 \\ 0.05 & 0.8 & 0.15 \\ 0 & 0.1 & 0.8 \end{bmatrix} \times \begin{bmatrix} p \\ q \\ 1-p-q \end{bmatrix} = \begin{bmatrix} p \\ q \\ 1-p-q \end{bmatrix} \xrightarrow[q =]{p =} \begin{bmatrix} 0.625 \\ 0.25 \\ 0.125 \end{bmatrix}$$



# * Markov Convergence Theorem

☑ Students

$$\begin{bmatrix} 0.8 & 0.25 \\ 0.2 & 0.75 \end{bmatrix} \times \begin{bmatrix} P \\ (1-P) \end{bmatrix} = \begin{bmatrix} P \\ (1-P) \end{bmatrix} \longrightarrow \frac{5}{9} = P$$

- Assumptions

  A1 : Finite states
  A2 : Fixed Transition Probabilities
  A3 : Can eventually get from any one state to any other
  A4 : Not a simple cycle

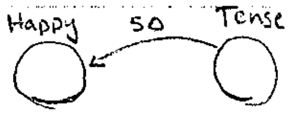
  — A3 means that it is possible that there is no direct transition between two states, but there should be a way between those two

- "Given A1~A4 a Markov process converges to an equilibrium distribution which is unque.

- The initial state doesn't matter !
- History doesn't matter !
- Intervening to change the state doesn't matter!

} Not in reality!

☑




~~this act change the system~~ temporarily but system tend to go back to the equlibrium.

- Intervention has no effect ?
  Does it mean that we should not have redistribution policy?

  — It could take a long time to get to equilibrium
  — what can we change? ⟹ transition probabilities ⟶ bring history and intervention into account

- Changing state ⟶ temporary
  Changing probs ⟶ permanant

we need interventions to change the probability ⟶ ☑ tipping points in history

☑ setting goal equilibrium

students example

20% alert ⟶ bored
25% bored ⟶ alert ⟹ how to change alert eq : 80% ?
alert eq : 55.56%

$$\begin{bmatrix} a & b \\ (1-a) & (1-b) \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \rightarrow 0.8a + 0.2b = 0.8$$

⟶ relation between alert-alert and bored ⟶ alert

$$0.8a + 0.2b = 0.8 \;\downarrow\; 8a + 2b = 8 \;\downarrow\; 4a + b = 4$$

## * Exaptation of Markov Model

- Markov Model = States + Transition Prob. → both fixed

☑ Voter Turnout

$V_t$ = voters @t

$N_t$ = non-voters @t

| | $V_t$ | $N_t$ |
|---|---|---|
| $V_{t+1}$ | 0.8 | 0.4 |
| $N_{t+1}$ | 0.2 | 0.6 |

→ # of people expected to vote in an election

people churn → distribution matters

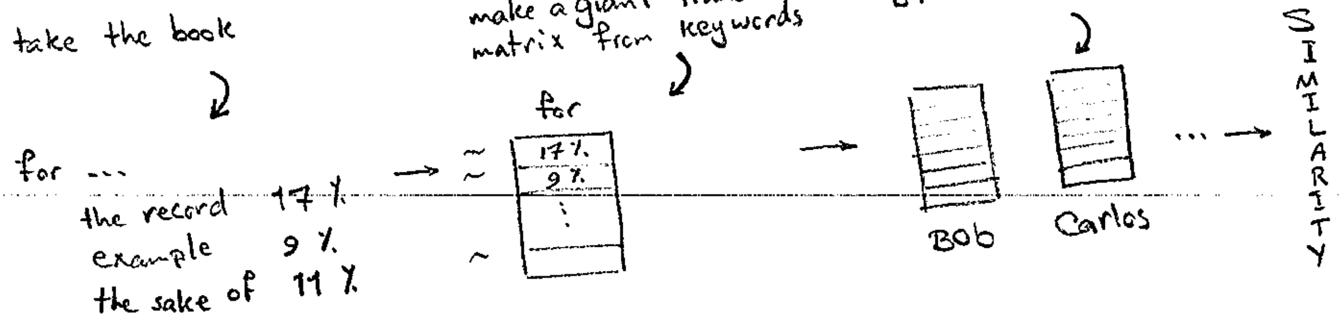
☑ School Enrollment

| | Go t | No t |
|---|---|---|
| $Go_{t+1}$ | 0.9 | 0.5 |
| $No_{t+1}$ | 0.1 | 0.5 |

→ total number of enrollment of student in school each day

- Expactation of Markov Transition Matrix
  - sth happens in time t and transitioned into sth at time t+1

☑ Identify Writers ~→ Arlene Saxonhouse




☑ Medical Diagnosis

do some treatments on patient by time → Success : pain → depression → pain → success

↳ drugs, diet regieme

failure : depression → mild → pain → no pain → fail

by looking on transition we can understand wether it's working or not.

☑ Lead Up to War

Two countries in tension : political statement → trade em bargoes → military build up

likelihood of having war according to history : 20%

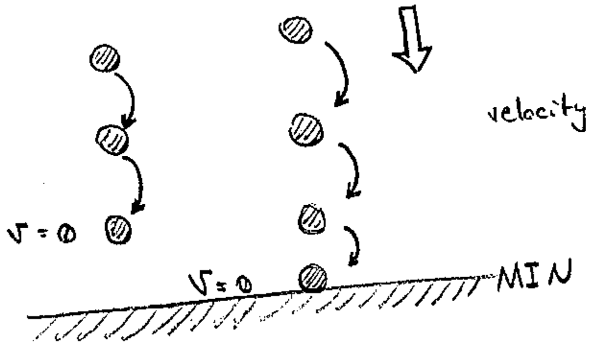
# * Lyapunov Function :

- Mapping models into outcomes
  - ↳ take a system / model → ask ourselves can I come with an Lyapunov function to describe that ?
  - ↳ If a Lyapunov func. found ⟹ that system goes to equilibrium for sure.
- States of a system : Equilibrium , Cycle , Random , Complex
  - ↳ if we can make a Lyapunov func. we can guarantee equilibrium state.
    - ↳ challenging
  - ↳ if we can't make it , we don't know → all 4 states possible

- Idea
  - ☑ velocity of a thing with these properties
    - ① if it move it falls down
    - ② there is a minimum level for falling

    conditions mean that the system has to stop.

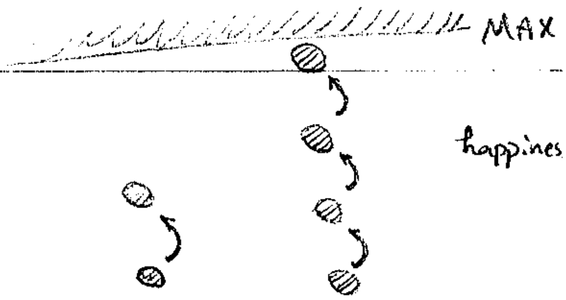
    Physics




  - ☑ happiness in trade
    - ① when people trade, happiness goes up (every time the sys. move happiness increases)
    - ② there is a maximum happiness

    when happiness hit the barrier → it stops → equilibrium

    Economics




- Formally
  - F(x) a Lyapunov function

    A1 : has a max value

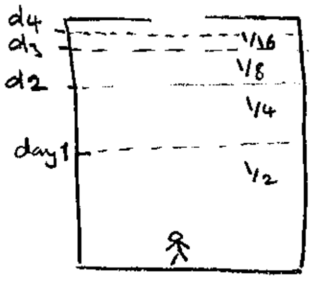
    A2 : $\exists\, k > 0 \implies x_{t+1} \neq x_t$ → if system moves

    $$F(x_{t+1}) > F(x_t) + k$$ → then increases at least by k
  - Claim : At some point $x_{t+1} = x_t$ → there is an equilibrium

- Zeno's Paradox
  - try to explain why "+k"
  - suppose I wanna leave this room , and I should cross the door

    First day I go half way → next day another half way → ...

    I never actually leave the room



- if I make my steps smaller in this fashion, I can never reach the maximum instead if I assume that each step is at least $1/16$ with max 16 steps I can go out.
  paradox is that you are actually moving toward that door but never reach it.
- Bonus: How fast? → if $k = 1/16$ I can leave the room with at most 16 steps
  the process gonna stop

- The HARD part → constructing the Lyapunov function.
- Formal: Physical explanation
  A1: has a minimum
  A2: $\exists k > 0$: if $x_{t+1} \neq x_t$, $F(x_{t+1}) < F(x_t) - k$
- Puzzle: Why the city looks self-organizing?
  - No central planner but no long queues for trains, grocery stores ...
    enough customers for resturants, ...

- C: cleaners
  G: grocery
  D: deli
  B: book store
  F: fish market
  } Places to go

  Monday
  Tuesday
  wednesday
  Thursday
  Friday
  } Days of week

  * Each day you have to decide during lunch hour where to go
  * Rule: avoid crowded places.

* Five random people
  (C, G, D, B, F)
  (G, C, D, B, F)
  (C, D, G, F, B)
  (C, B, F, G, D)
  (C, F, D, B, G)

* Behavior: People switch two locations so as to avoid crowds.

* Guy 1: go to C → 3 other ppl are there → goto F where 0 ppl are there and go to cleaners on day 5 when nobody is there.

* Lyapunov Function: Attemp 1 → Total number of ppl at each location
  ☒ 5 ppl go to C, 5 ppl goto D ... → that's not gonna work
* Attemp 2 → Total number of ppl that each person meet each week

| person 1 \ day | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| CGDBF | 3 | 0 | 2 | 2 | 1 | 8 |
| FGDBC | 0 | 0 | 2 | 2 | 0 | 4 |

if person 1 changes his schedule he meet 4 few ppl → those ppl will meet him 4 fewer times too → totally 8 fewer

↳ a Lyapunov func: peoples' rule is "switch so that I meet fewer people"
↳ minimum value = 0 & $k = 2$ if I don't meet fewer ppl (say 1) they don't meet me!

by keep switching it turns to a situation in which everybody meets noone 

(F, G, D, C, B)
(B, C, G, D, F)
(G, D, B, F, C)
(C, B, F, G, D)
(D, F, C, B, G)

* Exchange Market
• A1: Each person brings a wagon full of stuff
A2: People trade with others but only if each gets an increase in happiness by some amount K
– Attemp 1: Sum of happiness of people
↳ has max
↳ we only trade if some improvement happens ⟹ Lyapunov ✓

• NK: trades NW (nuke) for O (oil) ⟹ NK: happier ⟹ maybe the total happiness goes down
I: trades O for NW ⟹ I: happier ⟹ U do sth to compensate
U: not involved ⟹ U: less happy ⟹ system churns
we don't Know!

(north korea, iraq, usa)

• Political Coaliation
– Mergers ⟶ many role players, maybe not an equilibrium show up ⟶ sys. churns
– Alliances ⟶ same
• It depends on the size of externalities to have a Lyapunov Function
↳ race ⟶ going up for some, means going down for others ⟶ negative externality ⟶ hard to find Lyapunov
↳ altruism ⟶ going up for some, increase others too
• Related with lambda from simple cellular automata model
– systems that isn't influenced by others tend to go to equilibrium
– systems that my bahavior and actions are influenced by the others tend to be more complex or random

* How long until equilibrium?
Ex: $F(x_1) = 100$, $K = 2$, $Max = 200$ — # periods ≤ 50 | K = 4 ⟶ # periods ≤ 25
• We want to tighten the boundaries as much as possible
↳ decrease MAX
↳ increase K

☑ 100 pple → two waiting room
↘ switch the room if it's too busy → too busy threshold ≥ 58

Room A: 87 & Room B: 13 → max # of time periods to eq.?

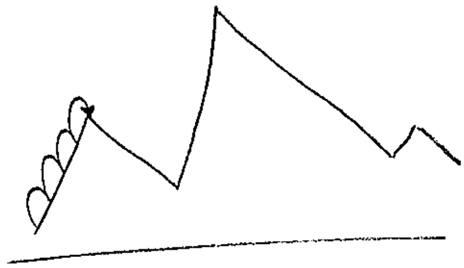
Maximum # of time period → K = min → k = 1 ⇒ 87 - 58 = 29 time periods

! changing room → happiness of person ↑
↘ happiness of room A ↑
↘ happiness of room B ↓ → externality → under threshold → no effect

* Does the process neccessarily reach MAX or MIN?

- N ☹ !

- Rugged Landscape



☑

① A > B > C
② B > C > A
③ C > A > B

→ person 1 has B
person 2 has C
person 3 has A
→ Exchange Market

person 1 & person 3 → P1: I have a "B" & I want your "A" → P3: no way!
Person 2 & person 1 → P2: I have a "C" & I want your "B" → P1: go to hell!
Person 3 & person 2 → P3: I have an "A" & I want your "C" → P2: forget it!

they can't get to the peak by pairwise trades
↳ everyone has his most desired

- It's possible to have the Lyapunov func. and have it stop before min/max.

* Does the system go to equilibrium?

☑ Chairs and Offices

↳ give each a person a chair → each person can trade their chair with someone else

↳ lyapunov function & have an eq.

↳ randomly assign ppl to offices → let them trade → there are externalities

→ I move to some place → some ppl move in for me
↘ some ppl move out for me

→ we don't know if it stops or not! → can we decide?

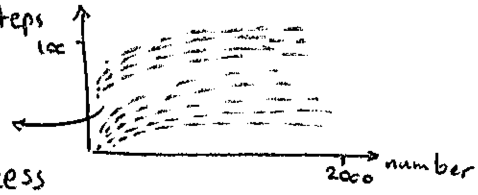
☑ Collatz Problem → HOTPO (Half Or Three Plus One)

{ pick a number
if even: divide by two
if odd: times 3 plus one
else if you reach one

* 5 → 16 → 8 → 4 → 2 → ①
* 7 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40
→ 20 → 10 → 5 → 16 → 8 → 4 → 2 → ①

draw the graph : number vs steps to stop

there is lots of structure in that graph




* Difference of Lyapunov func. with Markov Process

- States : equilibrium , cycle , random , complex
  - ↳ both of them could achieve it with certain conditions

- Lyapunov Func: A1 : minimum value
  A2 : $\exists k>0$, if $x_{t+1} \neq x_t \Rightarrow F(x_{t+1}) > F(x_t) + k$

- Markov Process: A1 : finite states
  A2 : fixed transition probabilities
  A3 : can eventually get from any one state to any other
  A4 : not a simple cycle
  - ↳ Stochastic equilibrium → system is churning but prob. is fixed
    - ex) A B C
      1/2 1/4 1/4
  - ↳ equilibrium is unique → history doesn't matter.
    → initial point " " "

---

- Lyapunov function
  - ↳ could be highly path dependent
  - ↳ could depend a lot on initial conditions
  - ↳ many equilibria is possible
  - ↳ fixed equilibrium (not stochastic) -
  - ↳ system stops

- If you can construct a Lyapunov function Then it goes to equilibrium.
  Then you can compute maximum time to eq.
  Then that eq. need not be unique or efficient.

The reason system won't go to an equilibria is Externalities
↳ pointing opposite

# * Coordination & Culture

- Culture : differences between countries, similarity within people of the country
  - ↳ cultural behavior: behaviors that doesn't make much sense for outsiders, but when viewed within culture, make a lot of sense
  - ↳ can be defined as { Differences between / Similarities within / Interesting behavior }

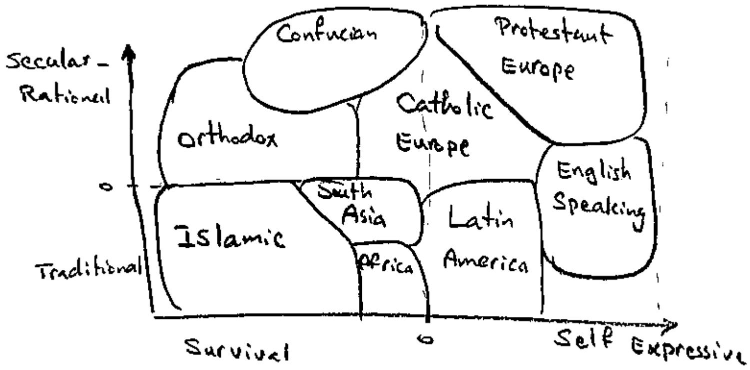
- Pure Coordination Game
  - → why people do the same thing
  - → why something they do is not optimal
  - → Does the system coordination happen? → Lyapunov function
  - → Many action coordination game → cultural differences → Axelrod's Model
    - → Coordination consistency

# * Culture

- what is culture?
  - ↳ Tylor (1871): complex whole which includes knowledges, belief, art, law, morals, customs
    - ↳ complex entirety of human existance that vary across the countries
  - ↳ Boaz (1911): totality of ~~mental and physical reactions~~ and activities that characterize behavioral responses to environment, others, and to himself
    - ↳ there should be consistency across the mental & physical reaction to the world
  - ↳ Trilling (1955)
    - → make a coherent life
    - → make possible the life of groups and individuals
    - → similarity within
  - ↳ all people don't do things alike → differences between
  - ↳ behaviors are interesting
    - → odd
    - → suboptimal
    - → don't have the functionality you'd like them to perform

- Ultimatum Game
  - Player 1 : offers split of 10.0 $ → give P1 ten dollar and he can split it as he wish
    Player 2 : accept or reject
    if accept → get split
    if reject → both get zero
  - P1 should figure out what is the min amount I should offer P2 to accept
  - People don't play it the same way
  - Lamalera : Indonesian whale hunters → more cooperation → offer 5.70 $
  - Machiguenga: Amazonian who lack personal name → offer 2.60 $

- Survey People
  - what are the relevant dimensions of different cultures
    - ↳ Dimension 1 : Survival Values ⟶ Self Expression Values
      - ☑ I have these cool glasses
    - ↳ Dimension 2 : Traditional Values ⟶ Secular - Rational Values
      - ☑ religeous
    - ↳ using factor analysis




- ↳ Each point is a representation of people cloud


- Hofstede's Dimensions
  - ↳ Power Distance ⟶ how much inequality are you willing to accept?
  - ↳ Uncertainty Avoidance
  - ↳ Individualism / Collectivism
  - ↳ Masculinity / Feminity
  - ↳ Confucian / Dynamism ⟶ how forward looking are you?

☑

| | USA | France | El Salvador | Korea |
|---|---|---|---|---|
| Power Dist | 32 | 61 | 62 | 56 |
| Individualism | 90 | 63 | 12 | 11 |
| Masculinity | 60 | 32 | 41 | 33 |
| Uncertaity Avoid. | 40 | 80 | 80 | 80 |

(El Salvador & Korea:) close according to numbers, far according to reality

- Why do we care?
  - Economy, political system, society ∝ culture
  - Commercial Transaction needs trust ⟶ economic backwardness due to lack of mutual confidence (Kenneth Arrow)
  - Social capital & trust must be "measurable" (Bob Solow)
  - Survey Trust
    - ↳ Indirect — cheat on tax, give away found money, ...
    - ↳ direct ⟶ do you think that most people can be trusted?
      - → Sweden 70%
      - → Italy 33%
      - → Turkey 10%

– Trust – GDP correlation
↳ not a pure correlation
but more of a causal relation

High GDP countries are high trust countries


## * Pure Coordination Game

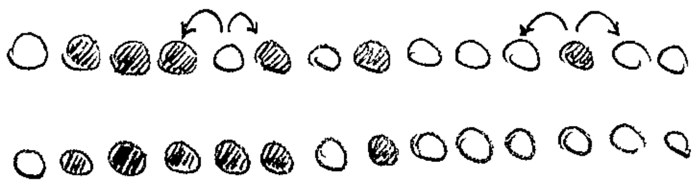
- The Ketchup Question
  - where you store your ketchup? fridge or cupboard?
  - ☑ USA → fridge     Britain → cupboard     ⇒ cultural decision
    you store ketchup where other people store their ketchup
    you want to put ketchup where your parents do → or else they can't find it!
  - ☑ Electrical Plug
  - ☑ Driving Direction
    ↳ failure to coordinate can be severe → Dagen H → In 3 Sep 1967 4:45 am Sweden changes the driving from left to driving from the right.

↳ some countries still drive on left → former British colonies & islands
↳ no border to change direction

- Formal Game
  - 2 players
  Action: they can put ketchup in fridge or cupboard.

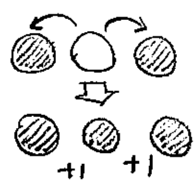
| | | P2: F | P2: C |
|---|---|---|---|
| P1 | F | 1 , 1 | 0 , 0 |
| | C | 0,0 | 1,1 |

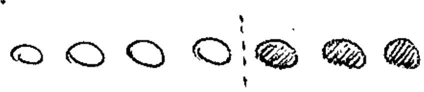
- N person Coordination Game
  ↳ people want to coordinate so they change behavior to match others.
  ↳ create similarities within



↳ does this process stop? or it keeps churning? → Lyapunov!
– Lyapunov function → A1: has a maximum
A2: $\exists\, k>0$, if $x_{t+1} \neq x_t \Rightarrow F(x_{t+1}) > F(x_t) + k$

↳ attemp: # of coordinations
k = 2 → each guy coordinates with two neighbors




↳ stops but not neccessarily when everyone is the same
☑ 

↳ depending on the social structure → everyone doing the exact same thing
↘ blocks of people doing very different behavior

- Coordination vs. Standing Ovation
  - Coord. game: Measurable difference in payoffs, no one would choose not to coordinate (you have to)
  - Standing ovation: could be more "psychological", it's okay to differ (you don't have to)

- Inefficient Coordination
  - Inefficient coordination game

☑ Maui – Des Moines game

Maui → hawaii; Des Moines → city in middle of Iowa

(P#2) (P#1)

| | M | D |
|---|---|---|
| M | (2, 2) | 0, 0 |
| D | 0, 0 | 1, 1 |

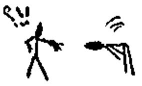
☑ Metric – English

Every body around me is using English units

| | M | E |
|---|---|---|
| M | 2, 2 | 0, 0 |
| E | 0, 0 | (1, 1) |

☑ Greetings

Shake – Bow Game



which is better? → { $a < 1$ → shake ✓ ; $a > 1$ → bow ✓ }

| | B | S |
|---|---|---|
| B | a, a | 0, 0 |
| S | 0, 0 | 1, 1 |

situation can shift → "a" is getting bigger because of disease.

☑ two ppl game
- choose randomly to shake or bow → if they don't agree they randomly choose again

with prob 1/2 they coordinate in one period, 1/4 in two. But it could take forever.

* Why Culture differ? why there is consistency within culture?

- We play more than one coordination game.
  - wear shoes inside the house
  - cross the street when "don't walk" sign is flashing
  - read newspaper at breakfast table
  - hug friends when see them
  - interrupt someone who is talking
- 20 coordination game → 2 answer each → over a million cultures!
- Axelrod's Culture Model
  - explains culture emergence, and how we see boundaries of cultures
  - Features : $\{1, 2, \dots, N\}$ → rules of coordination game
  - Traits : $a_i$ in $\{1, 2, 3, 4, 5, 6, 7\}$ → what action you take on that feature
  - Person : $(a_1, a_2, \dots, a_i, \dots, a_N)$ → person is a collection of traits over features

- Social space → contains people


- Assumption
  - ↳ assign values
  - ↳ pick a person
  - ↳ pick a neighbor
  - ↳ ask them "do you want to interact with this neighbor"?
    with probability P(similarity) pick a feature and match their traits
    - ↳ if the feature you choose is already matched it's OK, otherwise switch to match

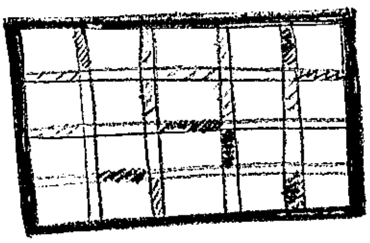
☑ 5 features
10 values
4 neighbor {N, S, E, W}
similarity = % traits that agree

Before Meeting: [ Leader | 5 | 3 | 2 | 1 | 1 | ; Follower | 5 | 1 | 3 | 3 | 1 | ] —prob. share 40%⟹ After Meeting: [ Follower | 5 | 3 | 3 | 1 | 1 | ]

Net logo Model → boundaries are black, the more similar the two neighbors the lighter the boundaries between them
↳ after convergance → different regions emerge
↳ thick boundaries around regions → or else they become similar to neighbors

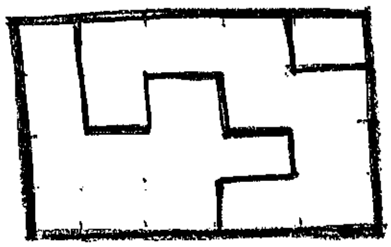
- Thick Boundaries: People near each other will either be exactly the same or differ by a lot.
  ↳ we get distinct regions in multidimension and their boundarie are self_reinforcing

initial



⟹

Convergance




- Bednar etal Model
  - coherence: consistency in behavior explained
  - heterogeneity within cultures explained
  - Assumptions:
    1. Values have "meaning"
    2. People desire consistency
    3. Innovation / Error

- Change the model to support Consistency as well as Coordinate
- Coordination rule :

Before Meeting :
Leader : | 5 | 3 | 2 | 1 | 1 |
Follower : | 5 | 1 | 3 | 3 | 1 |
⟹ Follower : | 5 | 3 | 3 | 3 | 1 |

- Consistency rule: pick two attributes, set the value of the second equal the value of the first.

| 5 | 1 | 4 | 4 | ⟹ | 5 | 5 | 4 | 4 |

* because the values have meanings, you behavior must be consistent

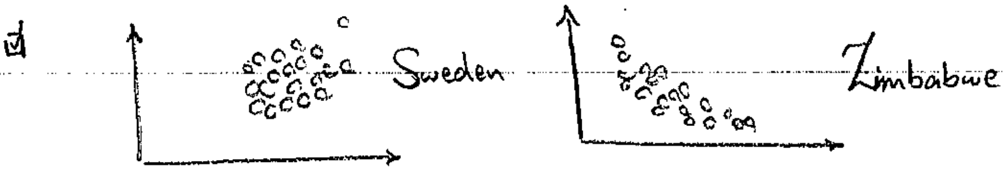
☑ family is not hugging, in college everyone hugs → start to hug in family

5 1     5 1     5 5

- Coordination + Consistency EXPECTED → takes long long time
  ↳ put in some errors and it converge!

- Small errors lead to substantial population level heterogeneity.
  ↳ little errors explains the cloud of behaviors in each country

☑




↳ why?

5 5 5 5 5
5 5 5 5 5
5 5 5 5 5
5 5 5 5 5

err ⟹ innovation

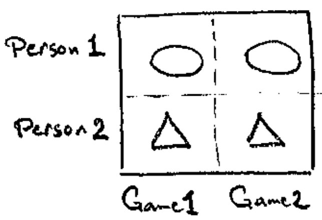
5 5 5 5 5
5 5 5 **6** 5
5 5 5 5 5
5 5 5 5 5

→ 6 turn into 5
→ other 5 become 6
→ other 5 become innovative e.g. 2

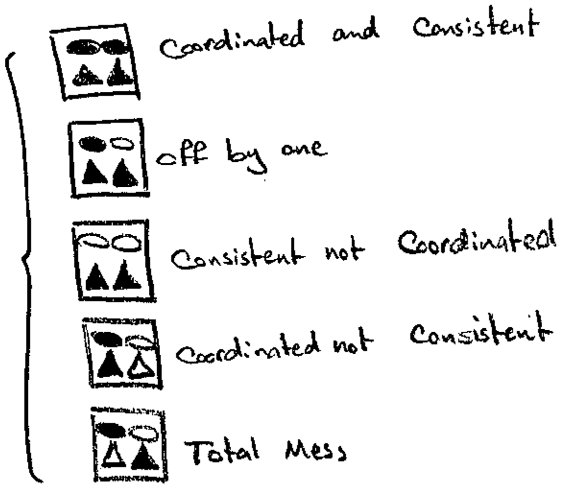
spread vertically : coordination → across people
spread horizontaly : consistency → in one person

↳ mathematical explaination with simplest model

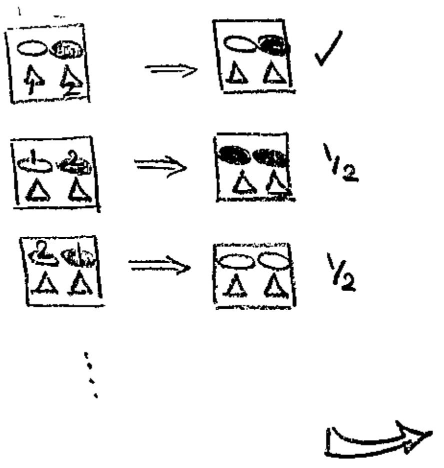
2 agent
2 games
2 actions

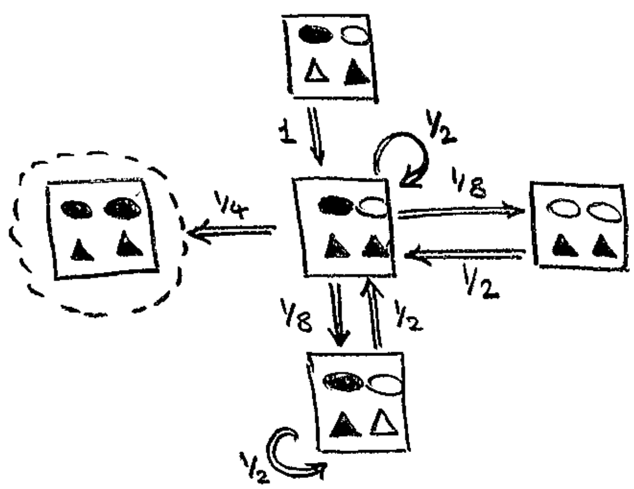



↳ States of the world



Consistency dynamics without error | Coordination & Consistency Transition Map:




↳ this transition map is different from Markov Process: we can't go from every state to another state

↳ if we allow errors we can get back from ( ) state to central state by prob. ε and now it is a Markov Process. so we can make a transition matrix for MP:

| t+1 \ t | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|
| S1 | $1-\varepsilon$ | $\varepsilon$ | 0 | 0 | 0 |
| S2 | $\frac{1}{4}$ | $\frac{1-\varepsilon}{2}$ | ... | | |
| S3 | | | | | |
| S4 | | | | | |
| S5 | | | | | |

* Small innovation (ε) leads to substantial heterogeneity

- Cultures
  - ↳ Difference between
  - ↳ Similarity within (not identical)
  - ↳ Interesting behavior → doesn't appear optimal from outside
  - ☑ 5 (5) 5
    - ↳ maybe this 5 is not optimal but because of consistency it is 5

* Summary
  - Cultural differences → coordination
    - ↘ coordinate on wrong action → idiosyncratically
      - ↘ or payoff changes over time
      - ↘ or in order to maintain consistency
  - Culture as multiple consistent coordination game
  - Small amount of innovation /error lead to withing culture heterogenity.
    - ↳ error propagates in 2 dimensions
  - Lyapunov and Markov models help us understand these processes.

# Path Dependence:

* Path dependence = what happen here now, depends on the path to get here
history matters

- Opposite of Markov Process
- Urn model → what cause p.d.?
  ↘ what really is p.d.?
  ↘ how to distinguish between different types of p.d.

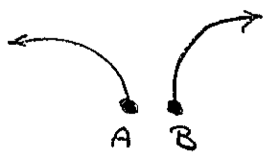
☑ Type writer keyboard → QWERTY
↘ there were initially lots of combinations of keys
→ QUERTY locked in because of increasing return

- Path Dependent: outcome probabilities depend upon the sequence of past outcomes

- "History Matters" → Technology
  ↘ Common Law
  ↘ Institiutional Choices
  ↘ Economic Success

- Increasing Return
  - Virtuous cycle → good building on good
  - not equal to path dependence

- Chaos
  - ESTIC: Extreme Sensitivity To Initial Conditions
  - chaos ≠ path dependence → chaos only cares for initial points




- Markov Process
  - Doesn't matter: path, initial point, history
  - sth in p.d. that violates Markov assumptions → answer: fixed transition probability
    ↳ in Urn model transition prob. change

* Urn Model
- Help us to distinguish → "path dependent" and "phat dependent"
  ↳ order doesn't matter
  ↳ the set of things matters
  ↳ processes that have outcomes in a given period that are p.d.
  ↳ processes that have long-run distributions over the sets of outcomes that are p.d. → have equilibrium

• Basic Urn Model

– Urn contains balls of various colors. The out come equals the color of ball selected.

↳ we calculate prob. of picking balls of different colors

– Bernoulli Model: most simple

↳ U = { Blue , Red }

↳ Select ball and return ⟶ fixed number of balls in urn

☑ Roulette

Black Jack ⟶ 16 cards of 52 are face cards

↳ Outcomes Independent ⟶ probability of each draw in each turn

☑ pick a red ball ⟶ doesn't have effect on prob. of picking red next time

↳ things that happens now doesn't depend in anyway on that happened in the past.

– Polya Process: most famous path dependent

↳ U = { 1× Blue , 1 × Red }

↳ Select ball and return

↳ Add a new ball that is the same color as the ball selected

↳ over time this prob. are gonna change

☑ 

Prob. of Red : $\frac{1}{2} \xrightarrow{red} \frac{2}{3} \xrightarrow{red} \frac{3}{4} \xrightarrow{blue} \frac{3}{5}$ ...

prob is changing over time and it is path dependent.

↳ Result 1: Any probability of red balls in an equilibrium and equally likely.

☑ you can end up with 60% red balls, 55% or 99% ⟶ they are all equally likely

equally likely to get 4% red balls or 80% !

☑ polya process as a model of fashion

Girl 1 : choice between a red shirt or a blue shirt ⟶ red

Girl 2 : look at her fried ⟶ 2 red , 1 blue ⟶ red

Girl 3 : what's more popular ⟶ 3 red , 1 blue ⟶ red ⟶ ...

any thing could happen!

↳ Result 2: Any sequence of events is equally likely.

☑ RRRB has the same probability of happening of BRRR

↳ If you know sth about the Frequency of Red and Blue balls, that wouldn't tell you anything about the order → any order is equally likely. P26

- Balancing Process: inverse of Polya Process
  - ↳ U: {1 × Blue, 1 × Red}
  - ↳ Select and Return
  - ↳ Add a ball that is the opposite color as the ball selected
    - ↳ this process is trying to balance it self out

☑

R: $\frac{1}{2}$, B: $\frac{1}{2}$ —red→ R: $\frac{1}{3}$, B: $\frac{2}{3}$ —red→ R: $\frac{1}{4}$, B: $\frac{3}{4}$ —blue→ R: $\frac{2}{5}$, B: $\frac{3}{5}$ ...

↳ Result 1: The balancing process converges to equal percentages of the two colors.

☑ where to have olympics?

Asia → Asia → Asia → now definitely want to put it somewhere else!

Balls of six different colors (Asia, Europe, Australia, Africa, N America, S America)

If you choose one ball, you should put balls of other colors to urn.

---

↳ Polya Process: Any probability with equal likelihood. → equilirium is really path depen

Balancing Process: 50% chance → not path dependent.

- Period Outcome: color of ball in a given period.
  Equilibrium: percentage of red balls in long run.

So...

Path dependent outcomes: color of ball in a given period depends on the path.

Path dependent equilibrium: percentage of red balls in long run depends on the path.

☑

| | P.D. Equilibria | P.D. Outcomes |
|---|---|---|
| Polya | ✓ | ✓ |
| Balancing | always $\frac{1}{2}$ ✗ | ✓ |

- History can matter at each moment but not have any impact in the long run.
  - ↳ no matter what happened in the way, the results (equilibria) is set (predictable) beforehanc

☑ Manifest Destiny → America is a country from sea to the shining sea.
No matter what happened in history: Louisiana Purchase, Wars, Gold rush, ...

☑ Railroads → they would have emerged, no matter in which order

- Path Dependent : outcome probabilities depend upon the <u>sequence</u> of past outcome.

Phat Dependent : outcome probabilities depend upon past outcomes but not their order.

- Polya Process is PHAT : outcome probabilities do not depend upon the order in which outcomes occured. they depend only on the set of outcomes that occured.
  - ↳ RRB, BRR ⟹ {2 Red, 1 Blue}

☑ Thirty rounds

Paths = $2 \times 2 \times 2 \times \cdots \times 2 = 2^{30}$ ⟶ different patterns

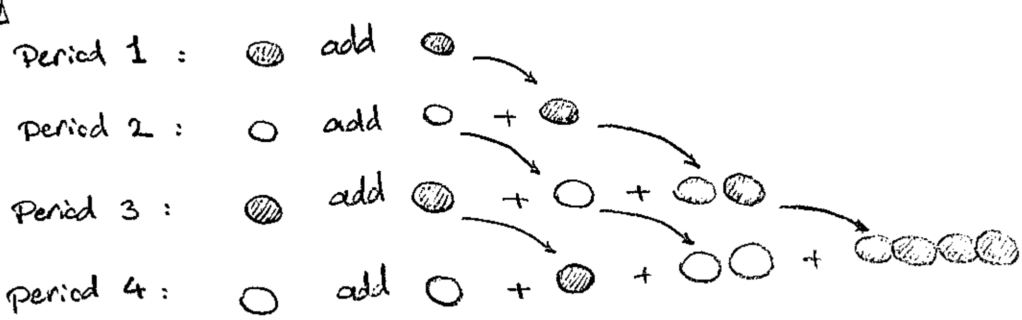
Sets = $\{0, 1, \ldots, 30\} = 31$ ⟶ number of blue balls can take 31 values

Path dependent ⟶ billions of patterns matter. ⟶ Is there any? Can Urn model make it? YES

Phat dependent ⟶ 31 set of outcomes matter.

- Sway Process : path dependent
  - ↳ U = {1× Blue , 1× Red}
  - ↳ Select and Return
  - ↳ In period t, add a ball of same color as selected ball and add $2^{t-s} - 2^{t-s-1}$ of color chosen in each period $(s<t)$.

☑




they call it sway: because as you go back in time decisions take on more and more weight.

↳ Path becomes more and more important, so you can get full path dependence.

☑ Law, Instituitional choices, Technological Adoptions ⟶ they all think about early movers having bigger effect.

↳ one way of having full path dependence is to put more and more weight on path as time passes.

• Polya Process Proof:

↳ Proof of Result 2: Any history of B blue and R red balls are equally likely.

$P(RBBB) = \frac{1}{2} \times \frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} = \frac{1\times1\times2\times3}{2\times3\times4\times5}$

$P(BBBR) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1\times2\times3\times1}{2\times3\times4\times5}$

* regardless of anything, denominator is: $2\times3\times4\times5$

* for 1 red ball → 1!
  for 3 blue ball → 3!

$P(RBRB) = \frac{1}{2} \times \frac{1}{3} \times \frac{2}{4} \times \frac{2}{5} = \frac{1\times1\times2\times2}{2\times3\times4\times5}$

$P(BBRR) = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \times \frac{2}{5} = \frac{1\times2\times1\times2}{2\times3\times4\times5}$

* for 2 red ball → 2!
  for 2 blue ball → 2!

because the order doesn't matter! ✓

↳ Proof of Result 1: Any probability of red balls is an equilibrium and equaly likely.

$P(BBBB) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{1}{5}$

$P(BBBR) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1}{20}$ → P(BBBR), P(BBRB), P(BRBB), P(RBBB) $\xrightarrow{\frac{1}{20}\times 5}$ $\frac{1}{5}$ of getting exactly 1 Re

~~odds~~ of getting 2 Reds, 3 Reds, and 4 Reds are $\frac{1}{5}$ too.

$P(k\times B) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{k-1}{k} \times \frac{k}{k+1} = \frac{1}{k+1} = P(k\times R)$

$P((k-1)\times B, 1\times R) = \frac{1}{2} \times \dots \times \frac{k-1}{k} \times \frac{1}{k+1} = \frac{1}{k\cdot(k+1)}$ $\xrightarrow{\text{Red going to be anywhere in } k \text{ slots}}$ $\frac{1}{k(k+1)} \times k = \frac{1}{k+1}$

$P((k-3)\times B, 3R) = \frac{1}{2} \times \dots \times \frac{k-3}{k-2} \times \frac{1}{k-1} \times \frac{2}{k} \times \frac{3}{k+1} = \frac{1\times2\times3}{(k-2)(k-1)k(k+1)}$ → $\frac{1}{k+1}$

ball R1 → k places
ball R2 → k-1 places
ball R3 → k-2 places
arranging 3 balls: 3x2x1

(k+1) possibility for having Red balls, all of them have probability $\frac{1}{k+1}$. ✓

* Difference with Markov Process

- Path dependent: outcome probabilities depend upon the sequence of past out comes.
- Markov process: A1: finite states
  A2: fixed transition probabilities
  A3: can eventually get from one state to another state
  A4: not a simple cycle

  ⟹ Unique Equilibrium (stochastic one) ↳ it is churning

In Markov process, according to A2, the history of events doesn't change probabilities.

• Effects of History → Markov Process: history doesn't matter if the prob.s don't change.

↘ Urn Model: history does matter if the prob.s change

## * Relation to Chaos

• Recursive Function:

↳ Outcome at time $t$ : $x(t)$

↳ Outcome function : $F: X \to X$

☑ $F(x) = x + 2$

☑ in Urn Model → outcome depends on current choice : $F(X_t) = X_{t+1}$

↘ outcome depends on all choices so far : $F(X_0, X_1, \ldots, X_t) = X_{t+1}$

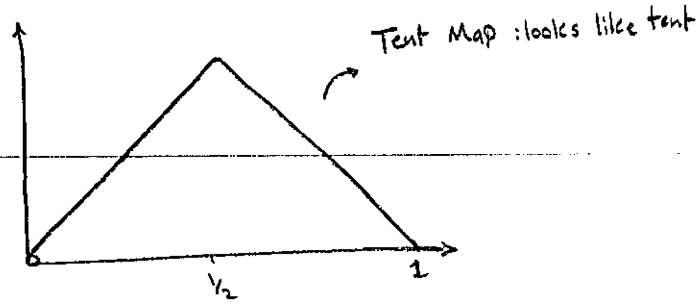
• Chaos → Extreme Sensitivity To Initial Conditions (ESTIC)

↳ If initial points $x$ and $x'$ differ by even a tiny amount, after many iterations of the outcome function, they differ by arbitary amount.

☑ Tent Map

$x \in (0, 1)$

$$F(x) = \begin{cases} 2x & \text{if } x < \frac{1}{2} \\ 2 - 2x & \text{if } x > \frac{1}{2} \end{cases}$$




$x = .21 \to .42 \to .84 \to .32 \to .64$

$x(1) = .4321 \to 0.8642 \to 0.2716 \to 0.5432 \to 0.9136 \to 0.1728 \to \ldots \to 0.9408 \to 0.1184 \to 0.2368$

$x(1) = .4321 \to 0.8642 \to 0.2716 \to 0.5432 \to 0.9136 \to 0.1728 \to \ldots \to 0.8544 \to 0.2912 \to 0.5824$

$x(2) = .4322 \to 0.8644 \to 0.2712 \to 0.5424 \to 0.9152 \to 0.1696 \to \ldots$

this is not path dependence, just initial points and fixed function.

• Independent process : outcome doesn't depend on starting point or what happens along the way

↳ probablistic concept

Chaos is deterministic → we know will happen once we get the initial point.

path dependent : outcome probabilities depend upon sequence of past outcomes.

Phat dependent : outcome probabilities depend upon past outcomes but not their order.

## * Increasing Returns.

• The more I have sth, the more other people want to have it → more produces more

☑ In Polya Process: The more blue balls I pick, the more likely I am to get a blue ball in future

↳ Is it increasing returns that cause P.d? ☑ Polya and Sway have increasing return. Balancing don't have it.

☑ Polya Process

↳ the more red balls I pick, the more likely I am to pick red balls in the future. ✓

- Is Increasing Return equivalent to Path Dependent Equilibrium?

— Gas / Electric Model

↳ U = {5 x Blue, 1 x Red} → Blue : gas car; → Red : electric car

↳ Select and Return

↳ If Red, add 1 Blue and 1 Red → an electric car has 1 gas engine & 1 electric one
If Blue, add 10 Blue

☑ 

Red: $\frac{1}{5}$ $\xrightarrow[\text{2 red}]{\text{6 blue}}$ Red: $\frac{2}{7}$ $\xrightarrow[\text{3 red}]{\text{7 blue}}$ Blue: $\frac{7}{10}$ $\xrightarrow[\text{3 red}]{\text{17 blue}}$ Blue: $\frac{17}{20}$ → ...

↳ Both have increasing return, but <u>eventually the blue will dominate</u>
↳ not p.d.

So this game is an example that these two concepts are not equal.

- Could you get Path Dependence without Increasing Returns?

— Symbiots

↳ U = {1 x Blue, 1 x Red, 1 x Green, 1 x Yellow}

↳ Select and Return

↳ If Red add 1 Green
If Green add 1 Red
If Blue add 1 Yellow
If Yellow add 1 Blue

↳ If we call Red and Green, G1 and Blue and Yellow G2 → Polya Process
↓
Path dependent

↳ This game doesn't have increasing return.

— Most of the time path dependence & Increasing Returns come together but they are logically two different concepts and can happen without each other.

* Externalities

- Situation that I make some choice A, and my dicision influences others.
  ↳ Interdependancy between choices.

☑ Public Projects → they are big → they likely bump into each other → create externalities
↳ Roads, Universities, Sewer Systems

- Model
  - ↳ Projects : {A, B, C, D, E}
  - ↳ Each has a value = 10
  - ↳ Each create externalities
  - ↳ Value Matrix

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 10 | | | | |
| B | -20 | 10 | | | |
| C | 5 | -10 | 10 | | |
| D | -10 | 30 | 0 | 10 | |
| E | 10 | -10 | 0 | 0 | 10 |

☑ first : (A) ⟶ 10

AB ⟶ 10 + 10 − 20 = 0 X

AC ⟶ 10 + 10 + 5 = 25 > A ⟶ (AC)

ACD ⟶ 10 + 10 + 5 + 10 − 10 ⟶ (Maybe!)

☑ first : (B) ⟶ 10

AB ⟶ 0 X

BC ⟶ 10 + 10 − 10 = 10 ⟶ (Maybe!)

BD ⟶ 10 + 10 + 30 = 50 ⟶ (BD)

---

↳ Depends on Initial Condition
Depends on Path

↳ Positive externalities creates less path dependence ⟶ becaus of increasing returns

↳ Big reason for P.D is externalites ⟶ Increasing Returns is just one of the externalitie

* Path Dependent and Tipping points

- Path Dependent Equilibrium : Percentage of red balls in long run depends on the path
- Direct Tips (Active) : Somebody choose an action that changes the probs. of things happening.
  - Tipping point is a single instance of time when equilibrium gonna change drastically.
  - Measuring Tips : Diversity Index, Entropy
- Path Dependence : things are moving from state to state.

Tipping Points : a singular action that changes the state drastically.

☑ Is Polya Process P.D. or T.P.?

Result 1 : prob of red balls in an equilibrium and equally likely.

Let's draw four balls from the process

Diversity Index : Red balls : 0, 1, 2, 3, 4 — each $1/5$ ⟶ $D.I. = \frac{1}{\sum P_i^2} = \frac{1}{5(\frac{1}{5})^2} = 5$

Suppose: First ball Red

↳ $P(\underline{R}RRR) = \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{3}$

↳ $P(\underline{R}RRB) = \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{1}{10}$ × 3 positions = $\frac{3}{10}$

↳ $P(\underline{R}RBB) = \frac{2}{3} \times \frac{1}{4} \times \frac{2}{5} = \frac{1}{15}$ × 3 positions $\approx \frac{2}{10}$

↳ $P(\underline{R}BBB) = \frac{1}{3} \times \frac{2}{4} \times \frac{3}{5} = \frac{1}{10}$

$\Rightarrow \begin{cases} 4R \rightarrow \frac{4}{10} \\ 3R \rightarrow \frac{3}{10} \\ 2R \rightarrow \frac{2}{10} \\ 1R \rightarrow \frac{1}{10} \end{cases} \Rightarrow$ D.I. = 3.33

Initially the diversity index was 5 but something happened in the path that made it 3.3. That is path dependence. This is not abrupt tip (e.g. 5 → 1.2)

# Networks :

* why they become important?

  - Internet → graph of data
  - More & More data on it → polarity of ideas, trends, ...
  - Logic of networks? → how do they come to be?
    → what rules do people use to form connections to other
  - Structure → what are the measures?
    → How do we compare one network to another?
    → what are the properties of network?
    → How many nodes and edges are there?
    → How connected they are together?
  - Function → what does that structure do?
    ☑ six degrees of seperation → no one set up the network for that, that thing just emerges

(Networks : An Introduction By : M.E.J. Newman)

  - Logic : How it forms → Structure : Measures → Function : what it does

---

* Structure

  - Nodes
  - Edges : directed or undirected

☑ Nodes : 50 United States    Edges : Adjacency (undirected) (two states connected by an edge if they share a border)

☑ Nodes : Students    Edges (directed)

# Prisoners' Dilemma and Collective Action: